MATHEMATICS

http://www.mathematics.jhu.edu/

Mathematics is a way of defining and solving problems by combining logic with insight, and by finding patterns and structure. At a most basic level, we use the abstract concept of “number” to understand what we observe, and we develop the method of “counting”; at a higher level, we use the language of “calculus” to understand motion, and we develop the methods of differentiation and integration. But mathematics is more than just computations with numbers and derivatives. Math is a way of thinking—an art that describes the abstract structure of logic, reason, and the scientific method.

The undergraduate program in the Department of Mathematics is intended both for students interested in preparing for graduate study and research in pure mathematics, and for students interested in using mathematics to pose and solve problems in the sciences, engineering, social sciences, or other areas. Undergraduate mathematics majors and minors will study:

- The foundations of analysis, which begins with the study of functions and their derivatives and integrals
- The fundamentals of advanced algebra, which is based on axiomatic systems involving operations of addition and multiplication in general settings
- Additional subjects such as geometry, probability, and topology
- Applications of mathematics to science and/or engineering.

The graduate program is designed primarily to prepare students for research and teaching in mathematics. It is naturally centered around the research areas of the faculty, which include algebraic geometry, algebraic number theory, data-intensive computation, geometric analysis, harmonic analysis, mathematical physics, partial differential equations, stochastic partial differential equations, and topology. The program can be supplemented in applied directions by courses in theoretical physics, computer science, mechanics, probability, and statistics offered in other departments of the Krieger School of Arts and Sciences and in the Department of Applied Mathematics and Statistics in the Whiting School of Engineering.

Facilities

The Mathematics Department resides in Krieger Hall on the Keyser Quad of the Homewood Campus. Adjacent to Krieger Hall, The University’s Milton S. Eisenhower Library has an unusually extensive collection of mathematics literature, including all the major research journals, almost all of which are also accessible electronically. The stacks are open to students. The department also has a useful reference library, the Philip Hartman Library. Graduate students share departmental offices, and study space can also be reserved in the university library. Graduate students may access the department’s Linux and Windows servers, as well as computers in graduate student offices. The department also hosts numerous research seminars, special lectures, and conferences throughout the academic year.

Programs

The Department of Mathematics offers bachelor’s, master’s, and doctoral degrees across a variety of programs. Undergraduates can elect to pursue a major or minor in mathematics; the major has an Honors designation available, as well as a four-year BA/MA program. Graduate students can pursue doctoral work in the department. We do not have a terminal master’s program at this time.

- Mathematics, Bachelor of Arts (https://e-catalogue.jhu.edu/arts-sciences/full-time-residential-programs/degree-programs/mathematics/mathematics-bachelor-arts/)
- Mathematics, Minor (https://e-catalogue.jhu.edu/arts-sciences/full-time-residential-programs/degree-programs/mathematics/mathematics-minor/)
- Mathematics, Bachelor of Arts/Master of Arts (https://e-catalogue.jhu.edu/arts-sciences/full-time-residential-programs/degree-programs/mathematics/mathematics-bachelor-arts-master/)
- Mathematics, PhD (https://e-catalogue.jhu.edu/arts-sciences/full-time-residential-programs/degree-programs/mathematics/mathematics-phd/)

For current course information and registration go to https://sis.jhu.edu/classes/

Courses

AS.110.100. Data Analytics Workshop. 1 Credit.
In this two-week pre-college program, students work in groups to construct and present a data analysis project which collects, organizes, cleanses, and visualizes a dataset of their choosing. Topics include exploratory data analysis, data visualization, probability distributions, data scraping and cleansing, the basics of hypothesis testing, and regression modeling. Students will primarily use Microsoft Excel. Programs like Octave (Matlab), and Octoparse, will also be introduced to help students learn the basics of data analytics.
Area: Quantitative and Mathematical Sciences

AS.110.102. College Algebra. 3 Credits.
This introductory course will create a foundational understanding of topics in Algebra. An emphasis will be on applications to prepare students for future courses like Precalculus or Statistics. After a review of elementary algebra concepts, topics covered include: equations and inequalities, linear equations, exponents and polynomials, factoring, rational expressions and equations, relations and functions, radicals, linear and quadratic equations, higher-degree polynomials, exponential, logarithmic, and rational functions.
Area: Quantitative and Mathematical Sciences

AS.110.105. Precalculus. 4 Credits.
This course provides students with the background necessary for the study of calculus. It begins with a review of the coordinate plane, linear equations, and inequalities, and moves purposefully into the study of functions. Students will explore the nature of graphs and deepen their understanding of polynomial, rational, trigonometric, exponential, and logarithmic functions, and will be introduced to complex numbers, parametric equations, and the difference quotient.
Area: Quantitative and Mathematical Sciences

AS.110.106. Calculus I (Biology and Social Sciences). 4 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Many applications to the biological and social sciences will be discussed.
Area: Quantitative and Mathematical Sciences

AS.110.107. Calculus II (For Biological and Social Science). 4 Credits.
Differential and integral Calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Applications to the biological and social sciences will be discussed, and the courses are designed to meet the needs of students in these disciplines. Recommended Course Background: Grade of C- or Better in AS.110.106 or AS.110.108, or a 5 on the AP AB exam.
Area: Quantitative and Mathematical Sciences

AS.110.108. Calculus I (Physical Sciences & Engineering). 4 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor's theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.
Area: Quantitative and Mathematical Sciences

AS.110.109. Calculus II (For Physical Sciences and Engineering). 4 Credits.
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor's theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines. Recommended Course Background: Grade of C- or Better in AS.110.106 or AS.110.108, or a 5 on the AP AB exam.
Area: Quantitative and Mathematical Sciences

AS.110.113. Honors Single Variable Calculus. 4 Credits.
This is an honors alternative to the Calculus sequences AS.110.106-AS.110.107 or AS.110.108-AS.110.109 and meets the general requirement for both Calculus I and Calculus II (although the credit hours count for only one course). It is a more theoretical treatment of one variable differential and integral calculus and is based on our modern understanding of the real number system as explained by Cantor, Dedekind, and Weierstrass. Students who want to know the "why's and how's" of Calculus will find this course rewarding. Previous background in Calculus is not assumed. Students will learn differential Calculus (derivatives, differentiation, chain rule, optimization, related rates, etc), the theory of integration, the fundamental theorem(s) of Calculus, applications of integration, and Taylor series. Students should have a strong ability to learn mathematics quickly and on a higher level than that of the regular Calculus sequences.
Area: Quantitative and Mathematical Sciences

AS.110.125. Introduction to Data Analysis. 4 Credits.
This course introduces students to important concepts in data analytics using a hands-on analysis through case studies. Students will learn how to gather, analyze, and interpret data to drive strategic and operational success. Students will explore how to clean and organize data for analysis and how to perform calculations using spreadsheets, SQL and R programming. Topics include the data lifecycle, probability, statistics, hypothesis testing, set theory, graphing, regression, and data ethics.
Area: Quantitative and Mathematical Sciences

AS.110.201. Linear Algebra. 4 Credits.
Vector spaces, matrices, and linear transformations. Solutions of systems of linear equations. Eigenvalues, eigenvectors, and diagonalization of matrices. Applications to differential equations.
Prerequisite(s): Grade of C- or better in AS.110.107 OR AS.110.109 OR AS.110.113 OR AS.110.202 OR AS.110.302, or a 5 on the AP BC exam.
Area: Quantitative and Mathematical Sciences

AS.110.202. Calculus III. 4 Credits.
Calculus of functions of more than one variable: partial derivatives, and applications; multiple integrals, line and surface integrals; Green's Theorem, Stokes' Theorem, and Gauss' Divergence Theorem.
Prerequisite(s): Grade of C- or better in AS.110.107 OR AS.110.109 OR AS.110.113 OR AS.110.201 OR AS.110.212 OR AS.110.302, or a 5 or better on the AP BC exam.
Area: Quantitative and Mathematical Sciences

AS.110.205. Mathematics of Data Science. 4 Credits.
This course is designed for students of all backgrounds to provide a solid foundation in the underlying mathematical, programming, and statistical theory of data analysis. In today's data driven world, data literacy is an increasingly important skill to master. To this end, the course will motivate the fundamental concepts used in this growing field. While discussing the general theory behind common methods of data science there will be numerous applications to real world data sets. In particular, the course will use Python libraries to create, import, and analyze data sets.
Area: Quantitative and Mathematical Sciences

AS.110.211. Honors Multivariable Calculus. 4 Credits.
This course includes the material in AS.110.202 with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or especially mathematics. AS.110.211-AS.110.212 used to be an integrated yearlong course, but now the two are independent courses and can be taken in either order.
Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)
Area: Quantitative and Mathematical Sciences

AS.110.212. Honors Linear Algebra. 4 Credits.
This course includes the material in AS.110.201 with additional applications and theory, and is recommended only for mathematically able students majoring in physical science, engineering, or mathematics who are interested in a proof-based version of linear algebra. This course can serve as an Introduction to Proofs (IP) course. Prerequisites: Grade of B+ or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam. Area: Quantitative and Mathematical Sciences.
Prerequisite(s): Grade of B+ or better in AS.110.107 or AS.110.109 or AS.110.113 or AS.110.202, or AS.110.302, or a 5 on the AP BC exam.
Area: Quantitative and Mathematical Sciences

AS.110.225. Problem Solving Lab. 2 Credits.
This course is an introduction to mathematical reason and formalism in the context of mathematical problem solving, such as induction, invariants, inequalities and generating functions. This course does not satisfy any major requirement, and may be taken more than once for credit It is primarily used as training for the William Lowell Putnam Mathematics Competition. Area: Quantitative and Mathematical Sciences.
Area: Quantitative and Mathematical Sciences

AS.110.275. Introduction to Probability. 4 Credits.
This course follows the actuarial Exam P syllabus and learning objectives to prepare students to pass the SOA/CAS Probability Exam. Topics include axioms of probability, discrete and continuous random variables, conditional probability, Bayes' theorem, Chebyshev's Theorem, Central Limit Theorem, univariate and joint distributions and expectations, loss frequency, loss severity and other risk management concepts. Exam P learning objectives and learning outcomes are emphasized. Recommended Course Background: Calculus II
Prerequisite(s): AS.110.107 OR AS.110.109
Area: Quantitative and Mathematical Sciences

AS.110.276. Introduction to Financial Mathematics. 4 Credits.
This course is designed to develop students' understanding of fundamental concepts of financial mathematics. The course will cover mathematical theory and applications including the time value of money, annuities and cash flows, bond pricing, loans, amortization, stock and portfolio pricing, immunization of portfolios, swaps and determinants of interest rates, asset matching and convexity. A basic knowledge of calculus and an introductory knowledge of probability is assumed.
Area: Quantitative and Mathematical Sciences

AS.110.301. Introduction to Proofs. 4 Credits.
This course will provide a practical introduction to mathematical proofs with the aim of developing fluency in the language of mathematics, which itself is often described as "the language of the universe." Along with a library of proof techniques, we shall tour propositional logic, set theory, cardinal arithmetic, and metric topology and explore "proof relevant" mathematics by interacting with a computer proof assistant. This course on the construction of mathematical proof will conclude with a deconstruction of mathematical proof, interrogating the extent to which proof serves as a means to discover universal truths and assessing the mechanisms by which the mathematical community achieves consensus regarding whether a claimed result has been proven.
Area: Quantitative and Mathematical Sciences

AS.110.302. Differential Equations and Applications. 4 Credits.
This is a course in ordinary differential equations (ODEs), equations involving an unknown function of one independent variable and some of its derivatives, and is primarily a course in the study of the structure of and techniques for solving ODEs as mathematical models. Specific topics include first and second ODEs of various types, systems of linear differential equations, autonomous systems, and the qualitative and quantitative analysis of nonlinear systems of first-order ODEs. Laplace transforms, series solutions and the basics of numerical solutions are included as extra topics.Prerequisites: Grade of C- or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam. Area: Quantitative and Mathematical Sciences.
Prerequisite(s): Grade of C- or better in AS.110.107 or AS.110.109 or AS.110.113 or AS.110.201 or AS.110.202 or AS.110.211 or AS.110.212, or a 5 on the AP BC exam.
Area: Quantitative and Mathematical Sciences

AS.110.303. The Mathematics of Politics, Democracy, and Social Choice. 4 Credits.
This course is designed for students of all backgrounds to provide a mathematical introduction tosocial choice theory, weighted voting systems, apportionment methods, and gerrymandering. Inthe search for ideal ways to make certain kinds of political decisions, a lot of wasted effort couldbe averted if mathematics could determine that finding such an ideal were actually possible in thefirst place. The course will analyze data from recent US elections as well as provide historicalcontext to modern discussions in politics, culminating in a mathematical analysis of the USElectoral College. Case studies, future implications, and comparisons to other governing bodiesoutside the US will be used to apply the theory of the course. Students will use Microsoft Excel toanalyze data sets. There are no mathematical prerequisites for this course.
Area: Quantitative and Mathematical Sciences
Writing Intensive

AS.110.304. Elementary Number Theory. 4 Credits.
The student is provided with many historical examples of topics, each of which serves as an illustration of and provides a background for many years of current research in number theory. Primes and prime factorization, congruences, Euler's function, quadratic reciprocity, primitive roots, solutions to polynomial congruences (Chevalley's theorem), Diophantine equations including the Pythagorean and Pell equations, Gaussian integers, Dirichlet's theorem on primes.
Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)
Area: Quantitative and Mathematical Sciences

AS.110.311. Methods of Complex Analysis. 4 Credits.
This course is an introduction to the theory of functions of one complex variable. Its emphasis is on techniques and applications, and it serves as a basis for more advanced courses. Functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions.
Prerequisite(s): Grade of C- or better in (AS.110.202 or AS.110.211)
Area: Quantitative and Mathematical Sciences

AS.110.365. Mathematical Foundations of AI Bias. 4 Credits.
At the end of this course students should be able to understand various sources of algorithmic bias; understand what types of bias can or cannot be addressed in a given data set; be able to reason over when different algorithms can be applied to a data set, and how they can be interpreted; take the outcomes of a given algorithm and reason about the bias of the output. Recommended Course Background: Vector calc, linear algebra, a suffiently advanced stats course, programming ability in R, matlab or python
Prerequisite(s): AS.110.201 OR AS.110.202 OR EN.553.310
Area: Quantitative and Mathematical Sciences

AS.110.375. Introduction to Mathematical Cryptography. 4 Credits.
An Introduction to Mathematical Cryptography is an introduction to modern cryptography with an emphasis on the mathematics behind the theory of public key cryptosystems and digital signature schemes. The course develops the mathematical tools needed for the construction and security analysis of diverse cryptosystems. Other topics central to mathematical cryptography covered are: classical cryptographic constructions, such as Diffie-Hellmann key exchange, discrete logarithm-based cryptosystems, the RSA cryptosystem, and digital signatures. Fundamental mathematical tools for cryptography studied include: primality testing, factorization algorithms, probability theory, information theory, and collision algorithms. A survey of important recent cryptographic innovations, such as elliptic curves, elliptic curve and pairing-based cryptography are included as well. This course is an ideal introduction for mathematics and computer science students to the mathematical foundations of modern cryptography.
Area: Quantitative and Mathematical Sciences

AS.110.401. Introduction to Abstract Algebra. 4 Credits.
An introduction to the basic notions of modern abstract algebra and can serve as as Introduction to Proofs (IP) course. This course is an introduction to group theory, with an emphasis on concrete examples, and especially on geometric symmetry groups. The course will introduce basic notions (groups, subgroups, homomorphisms, quotients) and prove foundational results (Lagrange's theorem, Cauchy's theorem, orbit-counting techniques, the classification of finite abelian groups). Examples to be discussed include permutation groups, dihedral groups, matrix groups, and finite rotation groups, culminating in the classification of the wallpaper groups.Prerequisites: Grade of C- or better in 110.201 or 110.212 Area: Quantitative and Mathematical Sciences.
Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)
Area: Quantitative and Mathematical Sciences

AS.110.405. Real Analysis I. 4 Credits.
This course is designed to give a firm grounding in the basic tools of analysis. It is recommended as preparation (but may not be a prerequisite) for other advanced analysis courses and may be taken as an Introduction to Proofs (IP) course. Topics include the formal properties of real and complex number systems, topology of metric spaces, limits, continuity, infinite sequences and series, differentiation, Riemann-Stieltjes integration. Prerequisites: Grade of C- or better in 110.201 or 110.212 and 110.202 or 110.211
Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)
Area: Quantitative and Mathematical Sciences

AS.110.406. Real Analysis II. 4 Credits.
This course continues AS.110.405 with an emphasis on the fundamental notions of modern analysis. Sequences and series of functions, Fourier series, equicontinuity and the Arzela-Ascoli theorem, the Stone-Weierstrass theorem, functions of several variables, the inverse and implicit function theorems, introduction to the Lebesgue integral.
Area: Quantitative and Mathematical Sciences

AS.110.407. Honors Complex Analysis. 4 Credits.
AS.110.407. Honors Complex Analysis. 4.00 Credits.This course is an introduction to the theory of functions of one complex variable for honors students. Its emphasis is on techniques and applications, and can serve as an Introduction to Proofs (IP) course. Topics will include functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions, as well as applications to number theory and harmonic analysis.Area: Quantitative and Mathematical Sciences.This is not an Introduction to Proofs course (IP) and may not be taken as a first proof-based mathematics course except at the discretion of the instructor. This course satisfies a core requirement of the mathematics major as a second analysis course, and is a core requirement for honors in the major.
Prerequisite(s): AS.110.405 OR AS.110.415
Area: Quantitative and Mathematical Sciences

AS.110.411. Honors Algebra I. 4 Credits.
An introduction to the basic notions of modern algebra for students with some prior acquaintance with abstract mathematics. Elements of group theory: groups, subgroups, normal subgroups, quotients, homomorphisms. Generators and relations, free groups, products, abelian groups, finite groups. Groups acting on sets, the Sylow theorems. Definition and examples of rings and ideals.
Prerequisite(s): Grade of C- or better in AS.110.212 OR AS.110.304 OR AS.110.113 OR AS.110.405 OR AS.110.415 OR AS.110.407 OR AS.110.413 OR AS.110.421
Area: Quantitative and Mathematical Sciences

AS.110.412. Honors Algebra II. 4 Credits.
This is a continuation of 110.411 Honors Algebra I. Topics studies include principal ideal domains, structure of finitely generated modules over them. Introduction to field theory. Linear algebra over a field. Field extensions, constructible polygons, non-trisectability. Splitting field of a polynomial, algebraic closure of a field. Galois theory: correspondence between subgroups and subfields. Solvability of polynomial equations by radicals. Prerequisites: Grade of C- or better in 110.201 or 110.212.Area: Quantitative and Mathematical Sciences.
Prerequisite(s): C- or better in AS.110.411
Area: Quantitative and Mathematical Sciences

AS.110.413. Introduction To Topology. 4 Credits.
Topological spaces, connectedness, compactness, quotient spaces, metric spaces, function spaces. An introduction to algebraic topology: covering spaces, the fundamental group, and other topics as time permits.
Prerequisite(s): Grade of C- or better in (AS.110.202 OR AS.110.211)
Area: Quantitative and Mathematical Sciences

AS.110.415. Honors Analysis I. 4 Credits.
This highly theoretical sequence in analysis is reserved for the most able students. The sequence covers the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics.
Prerequisite(s): Grade of B+ or better in 110.201 or B- or better in 110.212 and B+ or better in 110.202 or B- or better in 110.211.
Area: Quantitative and Mathematical Sciences

AS.110.416. Honors Analysis II. 4 Credits.
Lebesgue integration and differentiation. Elementary Hilbert and Banach space theory. Baire category theorem. Continuation of AS.110.415, introduction to real analysis.
Prerequisite(s): Grade of C- or better in AS.110.415
Area: Quantitative and Mathematical Sciences

AS.110.417. Partial Differential Equations. 3 Credits.
This course is aimed at a first exposure to the theory of Partial Differential Equations by examples. Basic examples of PDEs (Boundary value problems and initial value problems): Laplace equation, heat equation and wave equation. Method of separation of variables. Fourier series. Examples of wave equations in one and two dimensions. Sturm-Liouville eigenvalue problems and generalized Fourier series. Self-adjoint operators and applications to problems in higher dimensions. Nonhomogeneous PDEs. Green's functions and fundamental solution for the heat equation. Prerequisites:Calculus III. Recommended: 110.405 or 110.415.
Area: Quantitative and Mathematical Sciences

AS.110.421. Dynamical Systems. 4 Credits.
This is a course in the modern theory of Dynamical Systems. Topic include both discrete (iterated maps) and continuous (differential equations) dynamical systems and focuses on the qualitative structure of the system in developing properties of solutions. Topics include contractions, interval and planar maps, linear and nonlinear ODE systems including bifurcation theory, recurrence, transitivity and mixing, phase volume preservation as well as chaos theory, fractional dimension and topological entropy. May be taken as an Introduction to Proofs (IP) course.Prerequisites: Grade of C- or better in 110.201 or 110.212 OR 110.202 or 110.211 and 110.302 Area: Quantitative and Mathematical Sciences
Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 or AS.110.211) AND 110.302
Area: Quantitative and Mathematical Sciences

AS.110.422. Representation Theory. 4 Credits.
This course will focus on the basic theory of representations of finite groups in characteristic zero: Schur's Lemma, Mashcke's Theorem and complete reducibility, character tables and orthogonality, direct sums and tensor products. The main examples we will try to understand are the representation theory of the symmetric group and the general linear group over a finite field. If time permits, the theory of Brauer characters and modular representations will be introduced.
Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.401 OR AS.110.411)
Area: Quantitative and Mathematical Sciences

AS.110.433. Introduction to Harmonic Analysis and Its Applications. 3 Credits.
The course is an introduction to methods in harmonic analysis, in particular Fourier series, Fourier integrals, and wavelets. These methods will be introduced rigorously, together with their motivations and applications to the analysis of basic partial differential equations and integral kernels, signal processing, inverse problems, and statistical/machine learning.
Prerequisite(s): (AS.110.201 OR AS.110.212 OR EN.553.291) AND (AS.110.202 OR AS.110.211) AND (AS.110.405 OR AS.110.415)
Area: Quantitative and Mathematical Sciences

AS.110.435. Introduction to Algebraic Geometry. 4 Credits.
Algebraic geometry studies zeros of polynomials in several variables and is based on the use of abstract algebraic techniques, mainly from commutative algebra, for solving geometric problems about these sets of zeros. The fundamental objects of study are algebraic varieties which are the geometric manifestations of solutions of systems of polynomial equations. Algebraic geometry occupies a central place in modern mathematics and has multiple conceptual connections with diverse fields such as complex analysis, topology and number theory.This course aims to provide to an undergraduate student majoring in mathematics the fundamental background to approach the study of algebraic geometry by providing the needed abstract knowledge also complemented by several examples and applications.
Area: Quantitative and Mathematical Sciences

AS.110.439. Introduction To Differential Geometry. 4 Credits.
Theory of curves and surfaces in Euclidean space: Frenet equations, fundamental forms, curvatures of a surface, theorems of Gauss and Mainardi-Codazzi, curves on a surface; introduction to tensor analysis and Riemannian geometry; theorema egregium; elementary global theorems.
Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.100.211)
Area: Quantitative and Mathematical Sciences

AS.110.441. Calculus on Manifolds. 3 Credits.
This course provides the tools for classical three-dimensional physics and mechanics. This course extends these techniques to the general locally Euclidean spaces (manifolds) needed for an understanding of such things as Maxwell's equations or optimization in higher dimensional contexts, eg. in economics. The course will cover the theory of differential forms and integration. Specific topics include Maxwell's equations in terms of 4D Lorentz geometry, vector (in particular, tangent) bundles, an introduction to de Rham theory, and Sard's theorem on the density of regular values of smooth functions. The course is intended to be useful to mathematics students interested in analysis, differential geometry, and topology, as well as to students in physics and economics.
Area: Quantitative and Mathematical Sciences

AS.110.443. Fourier Analysis. 4 Credits.
An introduction to the Fourier transform and the construction of fundamental solutions of linear partial differential equations. Homogeneous distributions on the real line: the Dirac delta function, the Heaviside step function. Operations with distributions: convolution, differentiation, Fourier transform. Construction of fundamental solutions of the wave, heat, Laplace and Schrödinger equations. Singularities of fundamental solutions and their physical interpretations (e.g., wave fronts). Fourier analysis of singularities, oscillatory integrals, method of stationary phase.
Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)
Area: Quantitative and Mathematical Sciences

AS.110.445. Mathematical and Computational Foundations of Data Science. 3 Credits.
We will cover several topics in the mathematical and computational foundations of Data Science. The emphasis is on fundamental mathematical ideas (basic functional analysis, reproducing kernel Hilbert spaces, concentration inequalities, uniform central limit theorems), basic statistical modeling techniques (e.g. linear regression, parametric and non-parametric methods), basic machine learning techniques for unsupervised (e.g. clustering, manifold learning), supervised (classification, regression), and semi-supervised learning, and corresponding computational aspects (linear algebra, basic linear and nonlinear optimization to attack the problems above). Applications will include statistical signal processing, imaging, inverse problems, graph processing, and problems at the intersection of statistics/machine learning and physical/dynamical systems (e.g. model reduction for stochastic dynamical systems).
Area: Quantitative and Mathematical Sciences

AS.110.503. Undergraduate Research in Mathematics. 1 - 4 Credits.
You must submit an Independent Academic Work form to enroll in this course. The form can be accessed through SEAM.
Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

AS.110.586. Independent Study. 1 - 4 Credits.
You must submit an Independent Academic Work form to enroll in this course. The form can be accessed through SEAM.
Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

AS.110.587. DRP Independent Study. 1 Credit.
Directed Reading Program (DRP) Independent Study.
Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.
Area: Quantitative and Mathematical Sciences

AS.110.599. Independent Study. 1 - 3 Credits.
You must submit an Independent Academic Work form to enroll in this course. The form can be accessed through SEAM.
Area: Quantitative and Mathematical Sciences

AS.110.601. Algebra I. 4 Credits.
The first of a two semester algebra sequence to provide the student with the foundations for Number Theory, Algebraic Geometry, Representation Theory, and other areas. Topics include refined elements of group theory, commutative algebra, Noetherian rings, local rings, modules, and rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras.
Area: Quantitative and Mathematical Sciences

AS.110.602. Algebra II. 4 Credits.
The second of a two semester algebra sequence to provide the student with the foundations for Number Theory, Algebraic Geometry, Representation Theory, and other areas. Topics include refined elements of group theory, commutative algebra, Noetherian rings, local rings, modules, and rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras.
Area: Quantitative and Mathematical Sciences

AS.110.605. Real Analysis. 4 Credits.
This course covers the theory of the Lebesgue theory of integration in d-dimensional Euclidean space, and offers a brief introduction to the theory of Hilbert spaces. Closely related topics in real variables, such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, Lp classes, and various results about differentiation are examined in detail. applications are basic facts about convolution operators and Fourier series, including results for the conjugate function and the Hardy-Littlewood maximal function.
Area: Quantitative and Mathematical Sciences

AS.110.607. Complex Variables. 4 Credits.
Analytic functions of one complex variable. Topics include Cauchy integral theorems, residue theory, conformal mapping, harmonic functions, riemann mapping theorem, normal families. Other topics may include Mittag-Leffler theorem, Weierstrass factorization theorem, elliptic functions, Picard theorem, and Nevanlinna theory.

AS.110.608. Riemann Surfaces. 4 Credits.
Abstract Riemann surfaces. Examples: algebraic curves, elliptic curves and functions on them. Holomorphic and meromorphic functions and differential forms, divisors and the Mittag-Leffler problem. The analytic genus. Bezout's theorem and applications. Introduction to sheaf theory, with applications to constructing linear series of meromorphic functions. Serre duality, the existence of meromorphic functions on Riemann surfaces, the equality of the topological and analytic genera, the equivalence of algebraic curves and compact Riemann surfaces, the Riemann-Roch theorem. Period matrices and the Abel-Jacobi mapping, Jacobi inversion, the Torelli theorem. Uniformization (time permitting).

AS.110.615. Algebraic Topology I. 4 Credits.
Singular homology theory, cohomology and products, category theory and homological algebra, Künneth and universal coefficient theorems, Poincaré and Alexander duality theorems, Lefschetz fixed-point theorem, covering spaces and fundamental groups. Prerequsites: the equivalent of one semester in both Abstract Algebra and Real Analysis (specifically, point set topology).

AS.110.616. Algebraic Topology II. 4 Credits.
Higher homotopy groups, CW complexes, cellular homology and cohomology, spectral sequences and comparison theorems, graded homological algebra, fibrations, Serre and Eilenberg-Moore spectral sequence, Eilenberg-MacLane spaces, Steenrod algebra, spectra.
Area: Quantitative and Mathematical Sciences

AS.110.617. Number Theory I. 4 Credits.
Elements of advanced algebra and number theory. Possible topics for the year-long sequence include local and global fields, Galois cohomology, semisimple algebras, class field theory, elliptic curves, modular and automorphic forms, integral representations of L-functions, adelic geometry and function fields, fundamental notions in arithmetic geometry (including Arakelov and diophantine geometry).
Area: Quantitative and Mathematical Sciences

AS.110.618. Number Theory II. 4 Credits.
Topics in advanced algebra and number theory. Possible topics for the year-long sequence include local and global fields, Galois cohomology, semisimple algebras, class field theory, elliptic curves, modular and automorphic forms, integral representations of L-functions, adelic geometry and function fields, fundamental notions in arithmetic geometry (including Arakelov and diophantine geometry).

AS.110.619. Lie Groups and Lie Algebras. 4 Credits.
Lie groups and Lie algebras, classification of complex semi-simple Lie algebras, compact forms, representations and Weyl formulas, symmetric Riemannian spaces.
Area: Quantitative and Mathematical Sciences

AS.110.631. Partial Differential Equations I. 4 Credits.
This course is the first in the sequence about the general theory of PDEs. The beginning of the course will describe several important results of functional analysis which are instrumental for the study of PDEs: Hahn-Banach theorem, Uniform boundedness and closed graph theorems, reflexive spaces and weak topologies. The topics covered include: theory of Sobolev spaces. Harmonic functions and their properties. Weyl theorem. General Elliptic operators. Existence theory for elliptic boundary value problems. Lax-Milgram theorem. Dirichlet principle. Fine properties of solutions of elliptic equations such as maximum principles, Harnack principles, Sobolev and H\"older regularity.

AS.110.632. Partial Differential Equations II. 4 Credits.
This course is the second in the sequence about the general theory of PDEs. It focuses on time-dependent equations. It includes: Space-time Banach spaces. Linear 2nd order parabolic using Galerkin approximations. Linear 2nd order hyperbolic using Galerkin approximations. Semigroup theory and application to linear parabolic and hyperbolic equations. Nonlinear equations of Schr\"odinger/Wave type. Basic 2nd order semilinear wave equations. Systems of nonlinear hyperbolic equations. Conservation laws. Applications to equations coming from physics (Examples: Euler, Navier-Stokes, Einstein, etc...)

AS.110.633. Harmonic Analysis. 4 Credits.
Fourier multipliers, oscillatory integrals, restriction theorems, Fourier integral operators, pseudodifferential operators, eigenfunctions. Undergrads need instructor's permission.
Area: Quantitative and Mathematical Sciences

AS.110.637. Functional Analysis. 4 Credits.
This class will explore basic aspects of functional analysis, focusing mostly on normed vector spaces. This will include the Hahn-Banach and open mapping theorems, a discussion of strong and weak topologies, the theory of compact operators, and spaces of integrable functions and Sobolev spaces, with applications to the study of some partial differential equations. Prerequisite: Real Analysis

AS.110.643. Algebraic Geometry I. 4 Credits.
Introduction to affine varieties and projective varieties. Hilbert's theorems about polynomials in several variables with their connections to geometry. Abstract algebraic varieties and projective geometry. Dimension of varieties and smooth varieties. Sheaf theory and?some notions of cohomology. Applications of sheaves to?geometry;?e.g., theory of divisors, rudiments of scheme theory for the understanding of the Riemann-Roch theorem for curves and surfaces. Other topics may include Jacobian varieties, resolution of singularities, birational geometry on surfaces, schemes, connections with complex analytic geometry and topology.?

AS.110.644. Algebraic Geometry II. 4 Credits.
Introduction to affine varieties and projective varieties. Hilbert's theorems about polynomials in several variables with their connections to geometry. Abstract algebraic varieties and projective geometry. Dimension of varieties and smooth varieties. Sheaf theory and?some notions of cohomology. Applications of sheaves to?geometry;?e.g., theory of divisors, rudiments of scheme theory for the understanding of the Riemann-Roch theorem for curves and surfaces. Other topics may include Jacobian varieties, resolution of singularities, birational geometry on surfaces, schemes, connections with complex analytic geometry and topology.
Area: Quantitative and Mathematical Sciences

AS.110.645. Riemannian Geometry I. 4 Credits.
This course is a graduate-level introduction to foundational material in Riemannian Geometry. Riemannian manifolds, a smooth manifold equipped with a Riemannian metric. Topics include connections, geodesics, Jacobi fields, submanifold theory including the second fundamental form and Gauss equations, manifolds of constant curvature, comparison theorems, Morse index theorem, Hadamard theorem and Bonnet-Myers theorem.

AS.110.646. Riemannian Geometry II. 4 Credits.
This course covers more advanced topics in Riemannian geometry chosen at the instructors discretion. Possible topics include: minimal surface theory, geometric heat flows, harmonic mappings, Einstein manifolds, etc.

AS.110.653. Stochastic Differential Equations: An Introduction With Applications. 4 Credits.
This course is an introduction to stochastic differential equations and applications. Basic topics to be reviewed include Ito and Stratonovich integrals, Ito formula, SDEs and their integration. The course will focus on diffusion processes and diffusion theory, with topics include Markov properties, generator, Kolmogrov's equations (Fokker-Planck equation), Feynman-Kac formula, the martingale problem, Girsanov theorem, stability and ergodicity. The course will briefly introduce applications, with topics include statistical inference of SDEs, filtering and control.
Area: Quantitative and Mathematical Sciences

AS.110.710. What is... Seminar. 2 Credits.
This is a professional development course for graduate students, where they will learn, practice, or enhance their skills at giving math talks. The course will run in the format of a "What is... Seminar", where each week one of the participants will present a 1 hour talk on an accessible and relatively self-contained topic, titled What is (insert your math notion of choice). In preparation for their talk, students will meet with the instructor at least once, where they will receive guidance and detailed advice to help them give a great talk. Although the definition of a "great talk" is subjective, participants should be willing to follow the instructors' advice. Graduate students at any stage of their PhD are encouraged to attend, regardless of their experience giving talks.
Area: Quantitative and Mathematical Sciences

AS.110.712. Topics in Mathematical Physics. 3 Credits.
The course will be about scattering for Nonlinear Klein Gordon, Nonlinear Schroedinger and Nonlinear wave equations with variable coefficients in one space dimension. The ultimate goal is to show stability of stationary solutions, so called kinks, as well as moving soliton solutions for certain nonlinear equations of mathematical physics. Recently there has been a lot of activity in this area which uses some interesting math like spectral theory, estimates of Fourier Multipliers, twisted Fourier transform,... in order to study operators with variable coefficients.

AS.110.721. Topics In Homotopy Type Theory. 4 Credits.
Homotopy type theory (HoTT) is a new proposed foundation system for mathematics that extends Martin-Löf's dependent type theory with Voevodsky's univalence axiom. Dependent type theory is a formal system for constructive mathematics, in which a theorem is proven by constructing a term in the type that encodes its statement. In Homotopy type theory, types are thought of as spaces and terms as points in those spaces. A proof that two terms in a common type are equal is now interpreted as a path between two points in a space. In particular, types might have interesting higher homotopical structure, which can be thought of as revealing fundamental differences between two proofs of a common proposition. One advantage of this foundation system is its amenability to computer formalization, which this course will illustrate by introducing the computer proof assistant Agda.

AS.110.726. Topics in Analysis. 3 Credits.
The topics covered will involve the theory of calculus of Functors applied to Geometric problems like Embedding theory. Other related areas will be covered depending on the interest of the audience.
Area: Quantitative and Mathematical Sciences

AS.110.727. Topics in Algebraic Topology. 3 Credits.
The class will be specifically focused on topics related to -theory, and its connections to number theory and manifold theory.

AS.110.731. Topics in Geometric Analysis. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.

AS.110.733. Topics In Alg Num Theory. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.

AS.110.737. Topics in Algebraic Geometry. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.

AS.110.739. Topics in Analytic Number Theory. 3 Credits.
This course will be on functional analysis (applied to number theory) and Connes-Meyer's spectral interpretation of zeroes of Hecke L functions. Topics will include : adeles, ideles, bornologies, spectral theory, condensed/liquid modules à la Scholze-Clausen, Pontryagin duality and almost-periodic functions, Tate's thesis, Connes-Meyer's spectral interpretation. Relations with category theory, quantum mechanics, Bost-Connes systems and non-commutative geometry will be evoked. This course will be designed to be appealing for students from analysis or from algebra.

AS.110.741. Topics in Partial Differential Equations. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.

AS.110.749. Topics in Differential Geometry. 3 Credits.
In this class, we will study Aaron Naber and Jeff Cheeger's recent result on proving codimension four conjecture. We plan to talk about some early results of the structure on manifolds with lower Ricci bound by Cheeger and Colding. We will prove quantitative splitting theorem, volume convergence theorem, and the result that almost volume cone implies almost metric cone. Then we will discuss regularity of Einstein manifolds and the codimension four conjecture.
Area: Quantitative and Mathematical Sciences

AS.110.750. Topics in Representation Theory. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.
Area: Quantitative and Mathematical Sciences

AS.110.757. Topics in Stochastic Dynamical Systems. 3 Credits.
The course will present an introduction to stochastic dynamical systems and some applications in model reduction and data assimilation. The main focus will be on stability and ergodicity of stochastic dynamical systems, including stochastic differential equations driven by white and fractional noise, and their numerical approximations. We will then discuss model reduction, focusing on Mori-Zwanzig formalism and approximation of the generalized Langevin equation, and methods on the parametric inference of related stochastic systems. Data assimilation and stochastic control will also be briefly introduced.
Area: Quantitative and Mathematical Sciences

AS.110.771. Mathematics GTA Teaching Seminar. 3 Credits.
he goals of this seminar center on the preparedness for graduate students in mathematics to engage in classroom instructions for undergraduates at Johns Hopkins University. This seminar augments the teaching orientation provided to graduate students by the CER and Mathematics Department by addressing (1) teaching-techniques: student-centered inclusive teaching strategies, facilitating small group work, incorporating student ideas and student thinking into active hole class discussions, and choosing appropriate mathematical tasks, (2) opportunities for practice teaching in classrooms before their first assignment to TA for a course in scaffolded micro-teaching experiences and (3) preparing for the practice of and documentation of a reflective teaching practice necessary for success in their careers as mathematicians and educators.
Area: Quantitative and Mathematical Sciences

AS.110.773. Topics in Data Science. 3 Credits.
Topics covered will vary from year to year and are at the discretion of the instructor.
Area: Quantitative and Mathematical Sciences

AS.110.800. Independent Study-Graduates. 3 - 9 Credits.
This is an independent study course for students interested in a working with a professor on a specific topic.
Area: Quantitative and Mathematical Sciences

AS.110.801. Thesis Research. 10 - 20 Credits.
This is an independent study course for students working with their advisor toward the completion of their thesis.

AS.110.802. Graduate Student Research. 1 - 9 Credits.
Graduate level research on a topic chosen by the professor and student.

Cross Listed Courses

Applied Mathematics & Statistics

EN.553.738. High-Dimensional Approximation, Probability, and Statistical Learning. 3 Credits.
The course covers fundamental mathematical ideas for certain approximation and statistical learning problems in high dimensions. We start with basic approximation theory in low-dimensions, in particular linear and nonlinear approximation by Fourier and wavelets in classical smoothness spaces, and discuss applications in imaging, inverse problems and PDE's. We then introduce notions of complexity of function spaces, which will be important in statistical learning. We then move to basic problems in statistical learning, such as regression and density estimation. The interplay between randomness and approximation theory is introduced, as well as fundamental tools such as concentration inequalities, basic random matrix theory, and various estimators are constructed in detail, in particular multi scale estimators. At all times we consider the geometric aspects and interpretations, and will discuss concentration of measure phenomena, embedding of metric spaces, optimal transportation distances, and their applications to problems in machine learning such as manifold learning and dictionary learning for signal processing.

Applied Physics

EN.615.641. Mathematical Methods for Physics and Engineering. 3 Credits.
This course covers a broad spectrum of mathematical techniques essential to the solution of advanced problems in physics and engineering. Topics include ordinary and partial differential equations, contour integration, tabulated integrals, saddlepoint methods, linear vector spaces, boundary-value problems, eigenvalue problems, Green's functions, integral transforms, and special functions. Application of these topics to the solution of problems in physics and engineering is stressed. Prerequisite(s): Vector analysis and ordinary differential equations (linear algebra and complex variables recommended).

EN.615.765. Chaos and Its Applications. 3 Credits.
The course will introduce students to the basic concepts of nonlinear physics, dynamical system theory, and chaos. These concepts will be studied by examining the behavior of fundamental model systems that are modeled by ordinary differential equations and, sometimes, discrete maps. Examples will be drawn from physics, chemistry, and engineering. Some mathematical theory is necessary to develop the material. Practice through concrete examples will help to develop the geometric intuition necessary for work on nonlinear systems. Students conduct numerical experiments using provided software, which allows for interactive learning. Prerequisite(s): Mathematics through ordinary differential equations. Familiarity with MATLAB is helpful. Consult instructor for more information.

EN.615.769. Physics of Remote Sensing. 3 Credits.
This course exposes the student to the physical principles underlying satellite observations of Earth by optical, infrared, and microwave sensors, as well as techniques for extracting geophysical information from remote sensor observations. Topics will include spacecraft orbit considerations, fundamental concepts of radiometry, electromagnetic wave interactions with land and ocean surfaces and Earth's atmosphere, radiative transfer and atmospheric effects, and overviews of some important satellite sensors and observations. Examples from selected sensors will be used to illustrate the information extraction process and applications of the data for environmental monitoring, oceanography, meteorology, and climate studies.

EN.615.775. Physics of Climate. 3 Credits.
To understand the forces that cause global climate variability, we must understand the natural forces that drive our weather and our oceans. This course covers the fundamental science underlying the nature of the Earth's atmosphere and its ocean. This includes development of the basic equations for the atmosphere and ocean, the global radiation balance, description of oceanic and atmospheric processes, and their interactions and variability. Also included will be a description of observational systems used for climate studies and monitoring, fundamentals underlying global circulation, and climate prediction models. Prerequisite(s): Undergraduate degree in physics or engineering or equivalent, with strong background in mathematics through the calculus level.

Applied and Computational Mathematics

EN.625.603. Statistical Methods and Data Analysis. 3 Credits.
This course introduces statistical methods that are widely used in modern applications. A balance is struck between the presentation of the mathematical foundations of concepts in probability and statistics and their appropriate use in a variety of practical contexts. Foundational topics of probability, such as probability rules, related inequalities, random variables, probability distributions, moments, and jointly distributed random variables, are followed by foundations of statistical inference, including estimation approaches and properties, hypothesis testing, and model building. Data analysis ranging from descriptive statistics to the implementation of common procedures for estimation, hypothesis testing, and model building is the focus after the foundational methodology has been covered. Software, for example R-Studio, will be leveraged to illustrate concepts through simulation and to serve as a platform for data analysis. Prerequisite(s): Multivariate calculus.

EN.625.616. Optimization in Finance. 3 Credits.
Optimization models play an increasingly important role in financial decisions. This course introduces the student to financial optimization models and methods. We will specifically discuss linear, integer, quadratic, and general nonlinear programming. If time permits, we will also cover dynamic and stochastic programming. The main theoretical features of these optimization methods will be studied as well as a variety of algorithms used in practice. Prerequisite(s): Multivariate calculus and linear algebra. Course Note(s): Due to overlap in subject matter in EN.625.615 and EN.625.616, students may not receive credit towards the MS or post-master's certificate for both EN.625.615 and EN.625.616.

EN.625.633. Monte Carlo Methods. 3 Credits.
This course is an introduction to fundamental tools in designing, conducting, and interpreting Monte Carlo simulations. Emphasis is on generic principles that are widely applicable in simulation, as opposed to detailed discussion of specific applications and/or software packages. At the completion of this course, it is expected that students will have the insight and understanding to critically evaluate or use many state-of-the-art methods in simulation. Topics covered include random number generation, simulation of Brownian motion and stochastic differential equations, output analysis for Monte Carlo simulations, variance reduction, Markov chain Monte Carlo, simulation-based estimation for dynamical (state-space) models, and, time permitting, sensitivity analysis and simulation-based optimization. Course Note(s): This course serves as a complement to the 700-level course EN.625.744 Modeling, Simulation, and Monte Carlo. EN.625.633 Monte Carlo Methods and EN.625.744 emphasize different topics, and EN.625.744 is taught at a slightly more advanced level. EN.625.633 includes topics not covered in EN.625.744 such as simulation of Brownian motion and stochastic differential equations, general output analysis for Monte Carlo simulations, and general variance reduction. EN.625.744 includes greater emphasis on generic modeling issues (bias-variance tradeoff, etc.), simulation-based optimization of real-world processes, and optimal input selection.
Prerequisite(s): Linear algebra and a graduate-level statistics course such as EN.625.603 Statistical Methods and Data Analysis.

EN.625.641. Mathematics of Finance. 3 Credits.
This course offers a rigorous treatment of the subject of investment as a scientific discipline. Mathematics is employed as the main tool to convey the principles of investment science and their use to make investment calculations for good decision making. Topics covered in the course include the basic theory of interest and its applications to fixed-income securities, cash flow analysis and capital budgeting, mean-variance portfolio theory and the associated capital asset pricing model, utility function theory and risk analysis, derivative securities and basic option theory, and portfolio evaluation.
Prerequisite(s): Multivariate calculus and a course in probability and statistics (such as EN.625.603 Statistical Methods and Data Analysis).

EN.625.642. Mathematics of Risk, Options, and Financial Derivatives. 3 Credits.
The concept of options stems from the inherent human desire and need to reduce risks. This course starts with a rigorous mathematical treatment of options pricing, and related areas by developing a powerful mathematical tool known as Ito calculus. We introduce and use the well-known field of stochastic differential equations to develop various techniques as needed, as well as discuss the theory of martingales. The mathematics will be applied to the arbitrage pricing of financial derivatives, which is the main topic of the course. We treat the Black-Scholes theory in detail and use it to understand how to price various options and other quantitative financial instruments. Topics covered in the course include options strategies, binomial pricing, Weiner processes and Ito's lemma, the Black-Scholes-Merton Model, futures options and Black's Model, option Greeks, numerical procedures for pricing options, the volatility smile, the value at risk, exotic options, martingales and risk measures. Course Note(s): This class is distinguished from EN.625.641 Mathematics of Finance: Investment Science (formerly 625.439) and EN.625.714 Introductory Stochastic Differential Equations with Applications, as follows: EN.625.641 Mathematics of Finance: Investment Science gives a broader and more general treatment of financial mathematics, and EN.625.714 Introductory Stochastic Differential Equations with Applications provides a deeper (more advanced) mathematical understanding of stochastic differential equations, with applications in both finance and non-finance areas.
Prerequisite(s): Multivariate calculus, linear algebra and matrix theory (e.g., EN.625.609 Matrix Theory), and a graduate-level course in probability and statistics (such as EN.625.603 Statistical Methods and Data Analysis).

EN.625.695. Time Series Analysis. 3 Credits.
This course will be a rigorous and extensive introduction to modern methods of time series analysis and dynamic modeling. Topics to be covered include elementary time series models, trend and seasonality, stationary processes, Hilbert space techniques, the spectral distribution function, autoregressive/ integrated/moving average (ARIMA) processes, fitting ARIMA models, forecasting, spectral analysis, the periodogram, spectral estimation techniques, multivariate time series, linear systems and optimal control, state-space models, and Kalman filtering and prediction. Additional topics may be covered if time permits. Some applications will be provided to illustrate the usefulness of the techniques. Course Note(s): This course is also offered in the Department of Applied Mathematics and Statistics (Homewood campus) as EN.553.639.
Prerequisite(s): Graduate course in probability and statistics (such as EN.625.603 Statistical Methods and Data Analysis) and familiarity with matrix theory and linear algebra.

EN.625.714. Introductory Stochastic Differential Equations with Applications. 3 Credits.
The goal of this course is to give basic knowledge of stochastic differential equations useful for scientific and engineering modeling, guided by some problems in applications. The course treats basic theory of stochastic differential equations, including weak and strong approximation, efficient numerical methods and error estimates, the relation between stochastic differential equations and partial differential equations, Monte Carlo simulations with applications in financial mathematics, population growth models, parameter estimation, and filtering and optimal control problems. Prerequisite(s): Multivariate calculus and a graduate course in probability and statistics, as well as exposure to ordinary differential equations.

Artificial Intelligence

EN.705.741. Reinforcement Learning. 3 Credits.
This course will focus on both the theoretical and the practical aspects of designing, training, and testing reinforcement learning systems. The course begins with an examination of Markov decision processes (MDPs), which provide a sound mathematical basis for modeling and solving complex sequential decision problems. The more traditional analytical method for solving MDPs, dynamic programming, will be reviewed. We will then examine the major reinforcement learning approaches, such as Monte Carlo methods, temporal difference methods, policy gradient methods, and deep learning methods, comparing them as appropriate to dynamic programming techniques. Fundamental issues and limitations on the performance of reinforcement learning algorithms (e.g., the credit assignment problem, the exploration / exploitation tradeoff, on-policy learning versus off-policy learning, partial observability, and algorithm convergence properties) will be examined for each approach. Weekly exercises and discussion topics will reinforce and expand on the classroom material. In addition, students will gain practical experience during a semester-long project by programming, training, and testing various reinforcement learning algorithms.
Prerequisite(s): EN.625.638/EN.605.647 - Neural Networks or experience programming artificial neural networks in a high-level language.

Chemical & Biomolecular Engineering

EN.540.468. Introduction to Nonlinear Dynamics and Chaos. 3 Credits.
An introduction to the phenomenology of nonlinear dynamic behavior with emphasis on models of actual physical, chemical, and biological systems, involving an interdisciplinary approach to ideas from mathematics, computing, and modeling. The common features of the development of chaotic behavior in both mathematical models and experimental studies are stressed, and the use of modern data-mining tools to analyze dynamic data will be explored. Knowledge of Linear Algebra and Ordinary Differential Equations is a prerequisite (at an undergraduate level); Some computing experience is desirable. Emphasis will be placed on the geometric/visual computer-aided description and understanding of dynamics and chaos.
Prerequisite(s): Students may receive credit for only one of EN.553.473 OR EN.553.673 OR EN.540.468 OR EN.540.668.;((AS.110.201 OR AS.110.212) AND (AS.110.302 OR AS.110.306)) OR EN.553.291
Area: Engineering, Quantitative and Mathematical Sciences

EN.540.668. Introduction to Nonlinear Dynamics and Chaos. 3 Credits.
An introduction to the phenomenology of nonlinear dynamic behavior with emphasis on models of actual physical, chemical, and biological systems, involving an interdisciplinary approach to ideas from mathematics, computing, and modeling. The common features of the development of chaotic behavior in both mathematical models and experimental studies are stressed, and the use of modern data-mining tools to analyze dynamic data will be explored. Knowledge of Linear Algebra and Ordinary Differential Equations is a prerequisite (at an undergraduate level); Some computing experience is desirable. Emphasis will be placed on the geometric/visual computer-aided description and understanding of dynamics and chaos.
Prerequisite(s): Students may receive credit for only one of EN.553.473 OR EN.553.673 OR EN.540.468 OR EN.540.668.;((AS.110.201 OR AS.110.212) AND (S.110.302 OR AS.110.306) OR EN.553.291[C]
Area: Engineering, Quantitative and Mathematical Sciences

Computer Science

EN.601.442. Modern Cryptography. 3 Credits.
Modern Cryptography includes seemingly paradoxical notions such as communicating privately without a shared secret, proving things without leaking knowledge, and computing on encrypted data. In this challenging but rewarding course we will start from the basics of private and public key cryptography and go all the way up to advanced notions such as zero-knowledge proofs, functional encryption and program obfuscation. The class will focus on rigorous proofs and require mathematical maturity. [Analysis]
Prerequisite(s): Students may receive credit for only one of EN.600.442, EN.601.442, EN.601.642.;(EN.601.230 OR EN.601.231) AND (EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421)
Area: Engineering, Quantitative and Mathematical Sciences

EN.601.642. Modern Cryptography. 3 Credits.
Same material as 601.442, for graduate students. Modern Cryptography includes seemingly paradoxical notions such as communicating privately without a shared secret, proving things without leaking knowledge, and computing on encrypted data. In this challenging but rewarding course we will start from the basics of private and public key cryptography and go all the way up to advanced notions such as zero-knowledge proofs, functional encryption and program obfuscation. The class will focus on rigorous proofs and require mathematical maturity. [Analysis] Required course background: Probability & Automata/Computation Theory
Prerequisite(s): Students may receive credit for only one of EN.601.442 OR EN.601.642.
Area: Engineering, Quantitative and Mathematical Sciences

EN.605.615. Compiler Design with LLVM. 3 Credits.
The components of a compiler appear in every software application that handles input from an external source. This course shows how the components of a compiler are built and how they fit together to extract meaning from the input and how the data flows through the compiler's components to become useful to applications. Students will get practical experience in how to use the LLVM tools to build a complete compiler for a subset of the C++ programming language that can target almost any platform. Students will also get experience in developing a "Just In Time" component for an application that will accept code as input into the application while it is running, to be compiled and linked into the application so the application can execute it. Prerequisites: This course has no formal prerequisites, but experience with C++ is highly recommended because LLVM is written in C++, and therefore, all homework will be in C++, and this course is software homework intensive.

EN.605.617. Introduction to GPU Programming. 3 Credits.
This course will teach the fundamentals needed to utilize the ever-increasing power of the GPUs housed in the video cards attached to our computers. For years, this capability was limited to the processing of graphics data for presentation to the user. With the CUDA and OpenCL frameworks, programmers can develop applications that harness this power directly to search, modify, and quickly analyze large amounts of various types of data. Students will be introduced to core concurrent programming principles, along with the specific hardware and software considerations of these frameworks. In addition, students will learn canonical algorithms used to perform high-precision mathematics and data transformations. Class time will be split between lectures and hands-on exercises. There will be two individual projects in both CUDA and OpenCL programming, which will allow students to independently choose demonstrable goals, develop software to achieve those goals, and present the results of their efforts.

EN.605.621. Foundations of Algorithms. 3 Credits.
This follow-on course to data structures (e.g., EN.605.202) provides a survey of computer algorithms, examines fundamental techniques in algorithm design and analysis, and develops problem-solving skills required in all programs of study involving computer science. Topics include advanced data structures (red-black and 2-3-4 trees, union-find), recursion and mathematical induction, algorithm analysis and computational complexity (recurrence relations, big-O notation, NP-completeness), sorting and searching, design paradigms (divide and conquer, greedy heuristic, dynamic programming, amortized analysis), and graph algorithms (depth-first and breadth-first search, connectivity, minimum spanning trees, network flow). Advanced topics are selected from among the following: randomized algorithms, information retrieval, string and pattern matching, and computational geometry. Prerequisite(s): EN.605.202 Data Structures or equivalent. EN.605.203 Discrete Mathematics or equivalent is recommended. Course Note(s): The required foundation courses may be taken in any order but must be taken before other courses in the degree. Students can only earn credit for one of EN.605.620, EN.605.621, or EN.685.621.

EN.605.622. Computational Signal Processing. 3 Credits.
This course introduces algorithms and architectures for the analysis and processing of digital signals, taking the computer science perspective. It emphasizes computational complexity and efficiency and the design and implementation of computer algorithms for processing signals, designing digital filters, and effectively presenting and displaying information. Topics include signal analysis, discrete Fourier transform (definition, applications, and fast algorithms), convolution and correlation, spectral estimation and display, filter design, signal encoding/decoding, time-frequency analysis, Software Defined Radio (SDR), arithmetic computational complexity, and applications Background in signal processing and mathematics will be introduced as needed.
Prerequisite(s): EN.605.621 Foundations of Algorithms or equivalent background, some knowledge of complex numbers and linear algebra (vectors and matrices).

EN.605.626. Image Processing. 3 Credits.
Fundamentals of image processing are covered, with an emphasis on digital techniques. Topics include digitization, enhancement, segmentation, the Fourier transform, filtering, restoration, reconstruction from projections, and image analysis including computer vision. Concepts are illustrated by laboratory sessions in which these techniques are applied to practical situations, including examples from biomedical image processing. Prerequisite(s): Familiarity with Fourier transforms.

EN.605.633. Social Media Analytics. 3 Credits.
Today an immense social media landscape is being fueled by new applications, growth of devices (e.g., Smartphones and devices), and human appetite for online engagement. With a myriad of applications and users, significant interest exists in the obvious question, "How does one better understand human behavior in these communities to improve the design and monitoring of these communities?" To address this question a multidisciplinary approach that combines social network analysis (SNA), natural language processing, and data analytics is required. This course combines all these topics to address contemporary topics such as marketing, population influence, etc. There will be several small projects. Prerequisite(s): Knowledge of Python or R; matrix algebra.
Prerequisite(s): Computer Science majors need to complete foundation requirement first.;Foundation Prerequisites for Cybersecurity Majors:EN.605.621 AND EN.695.601 AND EN.695.641

EN.605.645. Artificial Intelligence. 3 Credits.
This is a foundational course in Artificial Intelligence. Although we hear a lot about machine learning, artificial intelligence is a much broader field with many different aspects. In this course, we focus on three of those aspects: reasoning, optimization, and pattern recognition. Traditionally, the first was covered under “Symbolic AI” or “Good Old Fashioned AI” and the latter two were covered under “Numeric AI” (or more specifically, “Connectionist AI” or “Machine Learning”). However, despite the many successes of machine learning algorithms, practitioners are increasingly realizing that complicated AI systems need algorithms from all three aspects. This approach falls under the ironic heading “Hybrid AI”.
In this course, the foundational algorithms of AI are presented in an integrated fashion emphasizing Hybrid AI. The topics covered include state space search, local search, example based learning, model evaluation, adversarial search, constraint satisfaction problems, logic and reasoning, expert systems, rule based ML, Bayesian networks, planning, reinforcement learning, regression, logistic regression, and artificial neural networks (multi-layer perceptrons). The assignments weigh conceptual (assessments) and practical (implementations) understanding equally.
Prerequisite(s): EN.605.621 Foundations of Algorithms. A working knowledge of Python programming is assumed as all assignments are completed in Python.

EN.605.646. Natural Language Processing. 3 Credits.
This course surveys the principal difficulties of working with written language data, the fundamental techniques that are used in processing natural language, and the core applications of NLP technology. Topics covered in the course include language modeling, text classification, labeling sequential data (tagging), parsing, information extraction, question answering, machine translation, and semantics. The dominant paradigm in contemporary NLP uses supervised machine learning to train models based on either probability theory or deep neural networks. Both formalisms will be covered. A practical approach is emphasized in the course, and students will write programs and use open source toolkits to solve a variety of problems. Course prerequisite(s): There are no formal prerequisite courses, although having taken any of EN.605.649 Introduction to Machine Learning, EN.605.744 Information Retrieval, or EN.605.645 Artificial Intelligence is helpful. Course note(s): A working knowledge of Python is assumed. While some of the assigned exercises can be done in any programming language, we will sometimes provide example code in Python, and many of the labs are best solved in Python.Course note(s): A working knowledge of Python is assumed. While some of the assigned exercises can be done in any programming language, we will sometimes provide example code in Python, and many of the labs are best solved in Python.

EN.605.647. Neural Networks. 3 Credits.
This course provides an introduction to concepts in neural networks and connectionist models. Topics include parallel distributed processing, learning algorithms, and applications. Specific networks discussed include Hopfield networks, bidirectional associative memories, perceptrons, feedforward networks with back propagation, and competitive learning networks, including self-organizing and Grossberg networks. Software for some networks is provided. Prerequisite(s): Multivariate calculus and linear algebra. Course Note(s): This course is the same as EN.625.638 Neural Networks.

EN.605.649. Introduction to Machine Learning. 3 Credits.
Analyzing large data sets (“Big Data”), is an increasingly important skill set. One of the disciplines being relied upon for such analysis is machine learning. In this course, we will approach machine learning from a practitioner’s perspective. We will examine the issues that impact our ability to learn good models (e.g., inductive bias, the curse of dimensionality, the bias-variance dilemma, and no free lunch). We will then examine a variety of approaches to learning models, covering the spectrum from unsupervised to supervised learning, as well as parametric versus non-parametric methods. Students will explore and implement several learning algorithms, including logistic regression, nearest neighbor, decision trees, and feed-forward neural networks, and will incorporate strategies for addressing the issues impacting performance (e.g., regularization, clustering, and dimensionality reduction). In addition, students will engage in online discussions, focusing on the key questions in developing learning systems. At the end of this course, students will be able to implement and apply a variety of machine learning methods to real-world problems, as well as be able to assess the performance of these algorithms on different types of data sets. Prerequisite(s): EN.605.202 – Data Structures or equivalent.
Prerequisite(s): EN.605.202 – Data Structures or equivalent, EN.605.621 – Foundations of Algorithms or EN.685.621 – Algorithms for Data Science or 705.621 – Introduction to Algorithms

EN.605.662. Data Visualization. 3 Credits.
This course explores the underlying theory and practical concepts in creating visual representations of large amounts of data. It covers the core topics in data visualization: data representation, visualization toolkits, scientific visualization, medical visualization, information visualization, flow visualization, and volume rendering techniques. The related topics of applied human perception and advanced display devices are also introduced. Prerequisite(s): Experience with data collection/analysis in data-intensive fields or background in computer graphics (e.g., EN.605.667 Computer Graphics) is recommended.

EN.605.671. Principles of Data Communications Networks. 3 Credits.
This course provides an introduction to the field of data communications and computer networks. The course covers the principles of data communications, the fundamentals of signaling, basic transmission concepts, transmission media, circuit control, line sharing techniques, physical and data link layer protocols, error detection and correction, data compression, common carrier services and data networks, and the mathematical techniques used for network design and performance analysis. Potential topics include analog and digital signaling; data encoding and modulation; Shannon channel capacity; synchronous and asynchronously transmission; RS232 physical layer interface standards; FDM, TDM, and STDM multiplexing techniques; inverse multiplexing; analog and digital transmission; V series modem standards; PCM encoding and T1 transmission circuits; LRC, VRC, and CRC error detection techniques; Hamming and Viterbi forward error correction techniques; character and bit-oriented protocols, information transparency, and BSC and HDLC data link layer protocols; Huffman, MNP5, and Lempel-Ziv-Welch data compression algorithms; circuit, message, packet, and cell switching techniques; public key and symmetric encryption algorithms, authentication, digital signature, and message digest techniques, secure e-mail, PGP, TSL/SSL, Kerberos, and IPsec security algorithms; Ethernet, Wi-Fi, and TCP/IP networks; reliability, availability, and queuing analysis performance techniques.

EN.605.716. Modeling and Simulation of Complex Systems. 3 Credits.
This multi-disciplinary course focuses on the application of modeling and simulation principles to complex systems. A complex system is a large-scale nonlinear system consisting of interconnected or interwoven parts (such as a biological organism, an ecological system, the economy, fluids or strongly-coupled solids). The subject is interdisciplinary with foundations in mathematics, nonlinear science, numerical simulations and statistical physics. The course begins with an overview of complex systems, followed by modeling techniques based on nonlinear differential equations, networks, and stochastic models. Simulations are conducted via numerical calculus, analog circuits, Monte Carlo methods, and cellular automata. In the course we will model, program, and analyze a wide variety of complex systems, including dynamical and chaotic systems, cellular automata, and iterated functions. By defining and iterating an individual course project throughout the term, students will gain hands-on experience and understanding of complex systems that arise from combinations of elementary rules. Students will be able to define, solve, and plot systems of linear and non-linear systems of differential equations and model various complex systems important in applications of population biology, epidemiology, circuit theory, fluid mechanics, and statistical physics.Course prerequisite(s): Knowledge of elementary probability and statistics and previous exposure to differential equations. Students applying this course to the MS in Bioinformatics should also have completed at least one Bioinformatics course prior to enrollment.Course note(s): This course may be counted toward a three-course concentration in Bioinformatics.

EN.605.728. Quantum Computation. 3 Credits.
Quantum computing is no longer a purely theoretical notion. The NSA and NIST are preparing to transition to quantum resistant cryptography. We have now entered the intermediate-scale quantum era, with near-term applications such as quantum machine learning being explored. Scalable quantum computers aren't here yet, but ongoing developments suggest they are on their way. This course provides an introduction to quantum computation for computer scientists: the focus is on algorithms rather than physical devices, and familiarity with quantum mechanics (or any physics at all) is not a prerequisite. Instead, pertinent aspects of the quantum mechanics formalism are developed as needed in class. The course begins with an introduction to the QM formalism. It then develops the abstract model of a quantum computer, and discusses how quantum algorithms enable us to achieve, for some problems, a significant speedup (in some cases an exponential speedup) over any known classical algorithm. This discussion provides the basis for a detailed examination of quantum integer factoring, quantum search, and other quantum algorithms. The course also explores quantum error correction, quantum teleportation, and quantum cryptography. It concludes with a glimpse at what the cryptographic landscape will look like in a world with quantum computers. Required work includes problem sets and a research project. Prerequisites: Some familiarity with linear algebra and with the design and analysis of algorithms or instructor permission.

EN.605.729. Formal Methods. 3 Credits.
All science requires mathematics. Formal methods used in developing computer systems are mathematically based techniques for describing system properties. These formal methods then can provide frameworks within which developers can specify, develop, verify, and prove systems in a systematic, rather than ad hoc manner. According to some researchers, the application of formal specification and verification methods could avoid disasters such as Heartbleed bug, Ariane 5 rocket explosion, and Therac-25 radiation therapy machine harms. This course is an introduction to the vast world of formal methods. The course starts with review of propositional logic, predicate logic, and covers set theoretic specification methods via Z, temporal specification via PTL, grammars, and logic based methods via Caml and Coq proof assistant. Each student will carry out an investigation of an existing formal verification system, applying it to a suitable problem of the student's choice. Among possible projects will be the formal verification of problem solutions such as designing a semaphore, designing a machine learning algorithm, a web interface, a test suite, a sophisticated data structure, or a theorem.

EN.605.755. Systems Biology. 3 Credits.
Systems biology is the study of complex biological systems using theoretical, mathematical, and computational tools and concepts. The advent of genomics, big data, and highpowered computing is allowing better understanding and elucidation of these systems. Central to systems biology is the development of computational models, based on sound statistics, which incorporate biological structures and networks, and can be informed and tested, with data on multiple scales. In this class, students will learn to develop and use different types of models of complex biological systems and how to test and perturb them. Students will learn basic biological system components and dynamics, as well as the data formats, sources, and modeling tools required to interrogate them. Tools will be used relating to functional genomics, evolution, biochemical systems, and cell biology. Students will utilize a model they have developed and available data from public repositories to investigate both a discovery-based project and a hypothesis based project. Prerequisite(s): Courses in molecular biology (EN.605.205 Molecular Biology for Computer Scientists or AS.410.602 Molecular Biology) and differential equations.Prerequisite(s): Courses in molecular biology (EN.605.205 Molecular Biology for Computer Scientists or AS.410.602 Molecular Biology) and differential equations.
Prerequisite(s): Courses in molecular biology (EN.605.205 Molecular Biology for Computer Scientists or AS.410.602 Molecular Biology) and differential equations.

Cybersecurity

EN.695.615. Cyber Physical Systems Security. 3 Credits.

The age of Cyber-Physical Systems (CPS) has officially begun. Not long ago, these systems were separated into distinct domains, cyber and physical. Today, the rigid dichotomy between domains no longer exists. Cars have programmable interfaces, Unmanned Aerial Vehicles (UAVs) roam the skies, and critical infrastructure and medical devices are now fully reliant on computer control. With the increased use of CPS and the parallel rise in cyber-attack capabilities, it is imperative that new methods for securing these systems be developed. This course will investigate key concepts behind CPS including: control systems, protocol analysis, behavioral modeling, and Intrusion Detection System (IDS) development. The course will be comprised of theory, computation, and projects to better enhance student learning and engagement . The course will begin with the mathematics of continuous and digital control systems and then shift the focus to the complex world of CPS, where both a general overview for the different domains (Industrial Control, Transportation, Medical Devices, etc.) and more detailed case studies will be provided . Students will complete a number of projects, both exploiting security vulnerabilities and developing security solutions for UAVs and industrial controllers. Several advanced topics will be introduced including behavioral analysis and resilient CPS.Course Notes: There are no prerequisite courses; however, students will encounter many concepts and technologies in a short period of time. Student should have a basic understanding of python programming, networking, matrices, and Windows and Linux operating systems.

EN.695.641. Cryptology. 3 Credits.

This course provides an introduction to the principles and practice of contemporary cryptography. It begins with a brief survey of classical cryptographic techniques that influenced the modern development of the subject. The course then focuses on more contemporary work: symmetric block ciphers and the Advanced Encryption Standard, public key cryptosystems, digital signatures, authentication protocols, and cryptographic hash functions. The course also provides an overview of quantum resistant cryptography and, as time permits, other recent developments such as homomorphic encryption. Complexity theory and computational number theory provide the foundation for much of the contemporary work in cryptology; pertinent ideas from complexity and number theory are introduced, as needed, throughout the course.

Data Science

EN.685.621. Algorithms for Data Science. 3 Credits.

This course provides a survey of computer algorithms, examines fundamental techniques in algorithm design and analysis, and develops problem-solving skills required in all programs of study involving data science. Topics include advanced data structures for data science (tree structures, disjoint set data structures), algorithm analysis and computational complexity (recurrence relations, big-O notation, introduction to complexity classes (P, NP and NP-completeness)), data transformations (FFTs, principal component analysis), design paradigms (divide and conquer, greedy heuristic, dynamic programming), and graph algorithms (depth-first and breadth-first search, ordered and unordered trees). Advanced topics are selected from among the following: approximation algorithms, computational geometry, data preprocessing methods, data analysis, linear programming, multi-threaded algorithms, matrix operations, and statistical learning methods. The course will draw on applications from Data Science. Course Prerequisite(s): EN.605.201 Introduction to Programming Using Java or equivalent. EN.605.203 Discrete Mathematics or equivalent is recommended. Course Note(s): This required foundation course must be taken before other 605.xxx courses in the degree. This course does not satisfy the foundation course requirement for Bioinformatics, Computer Science, or Cybersecurity. Students can only earn credit for one of EN.605.620, EN.605.621, or EN.685.621.

EN.685.648. Data Science. 3 Credits.

This course will cover the core concepts and skills in the interdisciplinary field of data science. These include problem identification and communication, probability, statistical inference, visualization, extract/transform/load (ETL), exploratory data analysis (EDA), linear and logistic regression, model evaluation and various machine learning algorithms such as random forests, k-means clustering, and association rules. The course recognizes that although data science uses machine learning techniques, it is not synonymous with machine learning. The course emphasizes an understanding of both data (through the use of systems theory, probability, and simulation) and algorithms (through the use of synthetic and real data sets). The guiding principles throughout are communication and reproducibility. The course is geared towards giving students direct experience in solving the programming and analytical challenges associated with data science. The assignments weight conceptual (assessments) and practical (labs, problem sets) understanding equally. Prerequisite(s): A working knowledge of Python scripting and SQL is assumed as all assignments are completed in Python.

Electrical and Computer Engineering

EN.525.605. Intermediate Electromagnetics. 3 Credits.

This course provides a background in engineering electromagnetics required for more advanced courses in the field. Topics include vector calculus, Poisson's and Laplace's equations, Vector potentials, Green's functions, magnetostatics, magnetic and dielectric materials, Maxwell's equations, plane wave propagation and polarization, reflection and refraction at a plane boundary, frequency-dependent susceptibility functions, transmission lines, waveguides, and simple antennas. Practical examples are used throughout the course.

EN.525.614. Probability & Stochastic Processes for Engineers. 3 Credits.
This course provides a foundation in the theory and applications of probability and stochastic processes and an understanding of the mathematical techniques relating to random processes in the areas of signal processing, detection, estimation, and communication. Topics include the axioms of probability, random variables, and distribution functions; functions and sequences of random variables; stochastic processes; and representations of random processes. Prerequisite(s): A working knowledge of multi-variable calculus, Fourier transforms, and linear systems theory.

EN.525.616. Communication Systems Engineering. 3 Credits.
In this course, students receive an introduction to the principles, performance and applications of communication systems. Students examine analog modulation/demodulation systems (amplitude - AM, DSB & SSB; and angle - PM & FM) and digital modulation/demodulation systems (binary and M-ary) in noise and interference. Sub-topics include filtering, sampling, quantization, encoding and the comparison of coherent & noncoherent detection techniques to improve signal-to-noise ratio (SNR) and bit error rate (BER) performance. Special topics and/or problems will be assigned that provide knowledge of how communication systems work from a system engineering viewpoint in real-world environments. Prerequisite(s): A working knowledge of Fourier transforms, linear systems, and probability theory. Basic working knowledge of MATLAB.

EN.525.625. Laser Fundamentals. 3 Credits.
This course reviews electromagnetic theory and introduces the interaction of light and matter with an emphasis on laser theory. A fundamental background is established, necessary for advanced courses in optical engineering. Topics include Maxwell’s equations, total power law, introduction to spectroscopy, classical oscillator model, Kramers-Kroenig relations, line broadening mechanisms, rate equations, laser pumping and population inversion, laser amplification, laser resonator design, and Gaussian beam propagation.
Prerequisite(s): EN.525.605 Intermediate Electromagnetics or equivalent.

EN.525.627. Digital Signal Processing. 3 Credits.
This course examines fundamental principles and applications of Digital Signal Processing. Introductory topics include linear, time-invariant systems, discrete-time convolution, and frequency-domain representations of discrete-time signals and systems. Sampling and quantization of continuous-time signals are covered. The Discrete Fourier Transform and efficient algorithms for its computation are studied in detail. The z-transform and its application to linear discrete-time systems analysis is studied. The design of digital filters using the windowing, equiripple, impulse invariance, and bilinear transformation methods is treated, along with the implementation of digital filter difference equations using canonical structures. MATLAB is utilized to demonstrate and implement Digital Signal Processing techniques.Prerequisite(s): A working knowledge of linear systems and Fourier analysis. Familiarity with MATLAB.

EN.525.645. Modern Navigation Systems. 3 Credits.
This course explores the use of satellite, terrestrial, celestial, radio, magnetic, and inertial systems for the real-time determination of position, velocity, acceleration, and attitude. Particular emphasis is on the historical importance of navigation systems; avionics navigation systems for high performance aircraft; the Global Positioning System; the relationships between navigation, cartography, surveying, and astronomy; and emerging trends for integrating various navigation techniques into single, tightly coupled systems.

EN.525.661. UAV Systems and Control. 3 Credits.
This hardware-supplemented course covers the guidance, navigation- and control principles common to many small fixed-wing and multirotor unmanned aerial vehicles (UAVs). Building on classical control systems and modeling theory, students will learn how to mathematically model UAV flight characteristics and sensors, develop and tune feedback control autopilot algorithms to enable stable flight control, and fuse sensor measurements using extended Kalman filter techniques to estimate the UAV position and orientation. Students will realize these concepts through both simulation and interaction with actual UAV hardware. Throughout the course, students will build a full 6-degree-of-freedom simulation of controlled UAV flight using MATLAB and Simulink. Furthermore, students will reinforce their UAV flight control knowledge by experimenting with tuning and flying actual open-source quadrotor UAVs. Prerequisite(s): Background in control systems (e.g., EN.525.609 Continuous Control Systems) and matrix theory along with a working knowledge of MATLAB. Experience using Simulink is desired. Existing familiarity with C programming language, electronics, and microcontrollers will be helpful but is not required.

EN.525.707. Error Control Coding. 3 Credits.
This course presents error-control coding with a view toward applying it as part of the overall design of a data communication or storage and retrieval system. Block, trellis, and turbo codes and associated decoding techniques are covered. Topics include system models, generator and parity check matrix representation of block codes, general decoding principles, cyclic codes, an introduction to abstract algebra and Galois fields, BCH and Reed-Solomon codes, analytical and graphical representation of convolutional codes, performance bounds, examples of good codes, Viterbi decoding, BCJR algorithm, turbo codes, and turbo code decoding.
Prerequisite(s): Background in linear algebra, such as EN.625.609 Matrix Theory; in probability, such as EN.525.614 Probability and Stochastic Processes for Engineers; and in digital communications, such as EN.525.616 Communication Systems Engineering. Familiarity with MATLAB or similar programming capability.

EN.525.721. Advanced Digital Signal Processing. 3 Credits.
The fundamentals of statistical signal processing are presented in this course. Topics include matrix factorizations and least squares filtering, optimal linear filter theory, classical and modern spectral estimation, adaptive filters, and optimal processing of spatial arrays.
Prerequisite(s): EN.525.614 Probability and Stochastic Processes for Engineers, EN.525.627 Digital Signal Processing, linear algebra, and familiarity with a scientific programming language such as MATLAB.

EN.525.724. Introduction to Pattern Recognition. 3 Credits.
This course focuses on the underlying principles of pattern recognition and on the methods of machine intelligence used to develop and deploy pattern recognition applications in the real world. Emphasis is placed on the pattern recognition application development process, which includes problem identification, concept development, algorithm selection, system integration, and test and validation. Machine intelligence algorithms to be presented include feature extraction and selection, parametric and non-parametric pattern detection and classification, clustering, artificial neural networks, support vector machines, rule-based algorithms, fuzzy logic, genetic algorithms, and others. Case studies drawn from actual machine intelligence applications will be used to illustrate how methods such as pattern detection and classification, signal taxonomy, machine vision, anomaly detection, data mining, and data fusion are applied in realistic problem environments. Students will use the MATLAB programming language and the data from these case studies to build and test their own prototype solutions.
Prerequisite(s): EN.525.614 Probability and Stochastic Processes for Engineers or equivalent. A course in digital signal or imageprocessing is recommended, such as EN.525.627 Digital Signal Processing, EN.525.619 Introduction to Digital Image and Video Processing, 525.643 Real-Time Computer Vision, or 525.746 Image Engineering.

EN.525.762. Introduction to Wavelets. 3 Credits.
This is an introductory course on wavelet analysis, with an emphasis on the fundamental mathematical principles and basic algorithms. We cover the mathematics of signal (function) spaces, orthonormal bases, frames, time-frequency localization, the windowed Fourier transform, the continuous wavelet transform, discrete wavelets, orthogonal and biorthogonal wavelets of compact support, wavelet regularity, and wavelet packets. It is designed as a broad introduction to wavelets for engineers, mathematicians, and physicists.Prerequisite: Competence with multivariable calculus, linear algebra, and a scientific programming language is required, as well as familiarity with Fourier transforms and signal processing fundamentals such as the discrete Fourier transform, convolutions, and correlations.

EN.525.770. Intelligent Algorithms. 3 Credits.
Intelligent algorithms are, in many cases, practical alternative techniques for tackling and solving a variety of challenging engineering problems. For example, fuzzy control techniques can be used to construct nonlinear controllers via the use of heuristic information when information on the physical system is limited. Such heuristic information may come, for instance, from an operator who has acted as a "human-in-the-loop" controller for the process. This course investigates several concepts and techniques commonly referred to as intelligent algorithms; discusses the underlying theory of these methodologies when appropriate; and takes an engineering perspective and approach to the design, analysis, evaluation, and implementation of Intelligent Systems. Fuzzy systems, genetic algorithms, particle swarm and ant colony optimization techniques, and neural networks are the primary concepts discussed in this course, and several engineering applications are presented along the way. Expert (rule-based) systems are also discussed within the context of fuzzy systems. An intelligent algorithms research paper must be selected from the existing literature, implemented by the student, and presented as a final project. Prerequisite(s): Student familiarity of system-theoretic concepts is desirable.

EN.525.776. Information Theory. 3 Credits.
Information theory concerns the fundamental limits for data compressibility and the rate at which data may be reliably communicated over a noisy channel. Course topics include measures of information, entropy, mutual information, Markov chains, source coding theorem, data compression, noisy channel coding theorem, error-correcting codes, and bounds on the performance of communication systems. Classroom discussion and homework assignments will emphasize fundamental concepts, and advanced topics and practical applications (e.g., industry standards, gambling/finance, machine learning) will be explored in group and individual research projects.
Prerequisite(s): EN.525.614 Probability and Stochastic Processes for Engineers or equivalent.

EN.525.780. Multidimensional Digital Signal Processing. 3 Credits.
The fundamental concepts of multidimensional digital signal processing theory as well as several associated application areas are covered in this course. The course begins with an investigation of continuous-space signals and sampling theory in two or more dimensions. The multidimensional discrete Fourier transform is defined, and methods for its efficient calculation are discussed. The design and implementation of two-dimensional non-recursive linear filters are treated. The final part of the course examines the processing of signals carried by propagating waves. This section contains descriptions of computed tomography and related techniques and array signal processing. Several application oriented software projects are required.
Prerequisite(s): EN.525.614 Probability and Stochastic Processes for Engineers and EN.525.627 Digital Signal Processing or equivalents. Knowledge of linear algebra and MATLAB is helpful.

Environmental Engineering and Science

EN.575.704. Applied Statistical Analysis and Design of Experiments for Environmental Applications. 3 Credits.
This course introduces statistical analyses and techniques of experimental design appropriate for use in environmental applications. The methods taught in this course allow the experimenter to discriminate between real effects and experimental error in systems that are inherently noisy. Statistically designed experimental programs typically test many variables simultaneously and are very efficient tools for developing empirical mathematical models that accurately describe physical and chemical processes. They are readily applied to production plant, pilot plant, and laboratory systems. Topics covered include fundamental statistics; the statistical basis for recognizing real effects in noisy data; statistical tests and reference distributions; analysis of variance; construction, application, and analysis of factorial and fractional-factorial designs; screening designs; response surface and optimization methods; and applications to pilot plant and waste treatment operations. Particular emphasis is placed on analysis of variance, prediction intervals, and control charting for determining statistical significance as currently required by federal regulations for environmental monitoring.Prerequisite: Undergraduate statistics is strongly recommended

Environmental Planning and Management

EN.575.608. Optimization Methods for Public Decision Making. 3 Credits.

This course is an introduction to operations research as applied in the public sector. Public sector operation research involves the development and application of quantitative models and methods intended to help decision makers solve complex environmental and socio-economic problems. The course material is motivated by real-world problems and is presented in an environmental engineering-relevant context. Such problems include air pollution control, water resources management, transportation planning, scheduling, resource allocation, facility location, and biological conservation. Emphasis is placed on skill development in the definition of problems, the formulation of models, and the application of solution methodologies. Methodologies covered in this course include linear programming, integer programming, multiobjective optimization, and dynamic programming.

Financial Mathematics

EN.555.642. Investment Science. 3 Credits.

This is the key introductory course for the financial mathematics program and introduces the major topics of investment finance. The investment universe, its context of markets, and the flow of global capital are introduced. Details of equities, interest, bonds, commodities, forwards, futures, and derivatives are introduced to varying degree. The concepts of deterministic cash flow stream, valuation, term structure theories, risk, and single- and multi-period random cash flows are presented. Here the neoclassical theory of finance is introduced including the topics of efficient markets, the risk-return twins leading to the mean variance Capital Asset Pricing Model (CAPM), the efficient frontier, the intertemporal models, and Arbitrage Pricing Theory (APT). Some introductory models of asset dynamics (including the binomial model), basic options theory, and elements of hedging are also included in this course. Course Note(s): This course is the same as EN.553.642 offered by through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

EN.555.644. Introduction to Financial Derivatives. 3 Credits.

This is the first of a two-course sequence devoted to the mathematical modeling of securities and the markets in which they are created and exchanged. The basic cash, hybrid, and derivative instruments are reviewed and set in a rigorous mathematical context. This includes equities, bonds, options, forwards, futures, and swaps, as well as their dealer, overthe-counter, and exchange environment. Models of the term structure of interest rates, spot rates and, the forward rate curve are treated; derived from cash instruments (e.g., bonds and interest rates like LIBOR) as well as from derivatives (such as Eurodollar futures and swaps). Principles of static, discrete, continuous and dynamic probabilistic models for derivative analysis (including the Weiner process, Ito's Lemma, and an introduction to risk-neutral valuation) are applied to develop the binomial tree approach to option valuation, the Black-ScholesMerton differential equation, and the Black-Scholes formulas for option pricing. Course Note(s): This course is the same as EN.553.644 offered by through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

EN.555.645. Interest Rate and Credit Derivatives. 3 Credits.

This is the second of a two-course sequence devoted to the mathematical modeling of securities and the markets in which they are created and exchanged. Focus turns to interest rate derivatives and the credit markets. The martingale approach to risk-neutral valuation is covered, followed by interest rate derivatives and models of the short rate process (including Heath, Jarrow & Morton and the Libor Market Model); analysis of bonds with embedded options and other interest rate derivatives (e.g., caps, floors, swaptions). Credit risk and credit derivatives, including copula models of time to default, credit default swaps, and a brief introduction to collateralized debt obligations will be covered. A major component of this course is computational methods. This includes data and time series analysis (e.g., estimation of volatilities), developing binomial and trinomial lattices and derivative analysis schemes, and numerical approaches to solving the partial differential equations of derivatives. Course Note(s): This course is the same as EN.553.645 offered through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

Prerequisite(s): EN.555.644 Introduction to Financial Derivatives

EN.555.646. Financial Risk Management and Measurement. 3 Credits.

This course applies advanced mathematical techniques to the measurement, analysis, and management of risk. The focus is on financial risk. Sources of risk for financial instruments (e.g., market risk, interest rate risk, credit risk) are analyzed; models for these risk factors are studied and the limitation, shortcomings, and compensatory techniques are addressed. Throughout the course, the environment for risk is considered, be it regulatory or social (e.g., Basel capital accords). A major component of the course are the Value at Risk (VaR) and Conditional VaR measures for market risk in trading operations, including approaches for calculating and aggregating VaR, testing VaR, VaR-driven capital for market risk, and limitations of the VaR-based approach. Asset Liability Management (ALM), where liquidity risk as well as market risk can affect the balance sheet, is analyzed. Here, models for interest rate, spread, and volatility risks are applied to quantify this exposure. Another major component of the course is credit risk. Sources of credit risk, how measured risk is used to manage exposure, credit derivatives, techniques for measuring default exposure for a single facility (including discriminant analysis and Mertonbased simulation), portfolio risk aggregation approaches (including covariance, actuarial, Merton-based simulation, macro-economic default model, and the macro-economic cashflow model - for structured and project finance). Finally, there is a brief introduction to concepts and tools that remain valid for large and extreme price moves, including the theory of copulas and their empirical testing and calibration. Course Note(s): This course is the same as EN.553.646 offered through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

EN.555.647. Quantitative Portfolio Theory & Performance Analysis. 3 Credits.
This course focuses on modern quantitative portfolio theory, models, and analysis. Topics include intertemporal approaches to modeling and optimizing asset selection and asset allocation; benchmarks (indexes), performance assessment (including Sharpe, Treynor, and Jenson ratios) and performance attribution; immunization theorems; alpha-beta separation in management, performance measurement, and attribution; Replicating Benchmark Index (RBI) strategies using cash securities/ derivatives; Liability-Driven Investment (LDI); and the taxonomy and techniques of strategies for traditional management (Passive, Quasi-Passive [Indexing] Semi-Active [Immunization & Dedicated] Active [Scenario, Relative Value, Total Return and Optimization]). In addition, risk management and hedging techniques are also addressed. Course Note(s): This course is the same as EN.553.647 offered through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

EN.555.648. Financial Engineering and Structured Products. 3 Credits.
This course focuses on structured securities and the structuring of aggregates of financial instruments into engineered solutions of problems in capital finance. Topics include the fundamentals of creating asset-backed and structured securities—including mortgage-backed securities (MBS), stripped securities, collateralized mortgage obligations (CMOs), and other asset-backed collateralized debt obligations (CDOs)—structuring and allocating cash-flows as well as enhancing credit; equity hybrids and convertible instruments; asset swaps, credit derivatives, and total return swaps; assessment of structure-risk interest rate-risk and credit-risk as well as strategies for hedging these exposures; managing portfolios of structured securities; and relative value analysis (including OAS and scenario analysis). Course Note(s): This course is the same as EN.553.648 offered through the full-time Applied Mathematics & Statistics department for the residence Master of Science in Engineering in Financial Mathematics.

First Year Seminars

AS.001.141. FYS: The Art of Mathematics. 3 Credits.
Mathematics is so much more that simply the language of science, or a set of techniques for solving quantitative-based problems. In fact, it is not a science at all, but an art, a construct of the imagination that not only provides structure to the reality of the world, but also gives form to anything and everything we can possibly imagine. Many of its fundamental principles and methods of employment are shared by artists of all types, from musicians to painters, sculptors, and poets. In this First-Year Seminar, we will explore these principles and methods shared by mathematicians and artists, like the notions of abstraction, metaphor, and pattern, the aesthetic quality both mathematicians and artists give to their work, the geometry of representation and visualization, the imagination as a tool of discovery and structure, and the use of mathematics in art, as well as the use of art in mathematics. Along the way, we will talk to artists and mathematicians, and hopefully visit the studios and galleries of each.
Area: Quantitative and Mathematical Sciences

AS.001.184. FYS: The Mathematics of Politics, Democracy, and Social Choice. 3 Credits.
This First-Year Seminar is designed for students of all backgrounds to provide a mathematical introduction to social choice theory, weighted voting systems, apportionment methods, and gerrymandering. In the search for ideal ways to make certain kinds of political decisions, a lot of wasted effort could be averted if mathematics could determine that finding such an ideal were actually possible in the first place. The seminar will analyze data from recent US elections as well as provide historical context to modern discussions in politics, culminating in a mathematical analysis of the US Electoral College. Case studies, future implications, and comparisons to other governing bodies outside the US will be used to apply the theory of the course. Students will use Microsoft Excel to analyze data sets. There are no mathematical prerequisites for this course.
Area: Quantitative and Mathematical Sciences

Mathematics

AS.110.433. Introduction to Harmonic Analysis and Its Applications. 3 Credits.
The course is an introduction to methods in harmonic analysis, in particular Fourier series, Fourier integrals, and wavelets. These methods will be introduced rigorously, together with their motivations and applications to the analysis of basic partial differential equations and integral kernels, signal processing, inverse problems, and statistical/ machine learning.
Prerequisite(s): (AS.110.201 OR AS.110.212 OR EN.553.291) AND (AS.110.202 OR AS.110.211) AND (AS.110.405 OR AS.110.415)
Area: Quantitative and Mathematical Sciences

AS.110.445. Mathematical and Computational Foundations of Data Science. 3 Credits.
We will cover several topics in the mathematical and computational foundations of Data Science. The emphasis is on fundamental mathematical ideas (basic functional analysis, reproducing kernel Hilbert spaces, concentration inequalities, uniform central limit theorems), basic statistical modeling techniques (e.g. linear regression, parametric and non-parametric methods), basic machine learning techniques for unsupervised (e.g. clustering, manifold learning), supervised (classification, regression), and semi-supervised learning, and corresponding computational aspects (linear algebra, basic linear and nonlinear optimization to attack the problems above). Applications will include statistical signal processing, imaging, inverse problems, graph processing, and problems at the intersection of statistics/machine learning and physical/dynamical systems (e.g. model reduction for stochastic dynamical systems).
Area: Quantitative and Mathematical Sciences

AS.110.653. Stochastic Differential Equations: An Introduction With Applications. 4 Credits.
This course is an introduction to stochastic differential equations and applications. Basic topics to be reviewed include Ito and Stratonovich integrals, Ito formula, SDEs and their integration. The course will focus on diffusion processes and diffusion theory, with topics include Markov properties, generator, Kolmogrov's equations (Fokker-Planck equation), Feynman-Kac formula, the martingale problem, Girsanov theorem, stability and ergodicity. The course will briefly introduce applications, with topics include statistical inference of SDEs, filtering and control.
Area: Quantitative and Mathematical Sciences

Mechanical Engineering

EN.535.621. Intermediate Fluid Dynamics. 3 Credits.

This course prepares the student to solve practical engineering flow problems and concentrates on the kinematics and dynamics of viscous fluid flows. Topics include the control volume and differential formulations of the conservation laws, including the Navier-Stokes equations. Students examine vorticity and circulation, dynamic similarity, and laminar and turbulent flows. The student is exposed to analytical techniques and experimental methods, and the course includes an introduction to computational methods in fluid dynamics. It also includes a programming project to develop a numerical solution to a practical fluid flow problem. Prerequisite(s): An undergraduate fluid mechanics course.

EN.535.735. Computational Fluid Dynamics. 3 Credits.

This is a three-branch course covering theory, implementation, and application of computational fluid dynamics (CFD). The theory side covers the basics of CFD, finite volume discretization schemes, time integration, solution of systems of equations, boundary conditions, error analysis and turbulence models. On the implementation side students will implement a number of small-scale CFD solvers and pre-processing tools in order to get a working knowledge of the simulation process. The application side covers the use of a fully featured, readily available CFD solver to study an array of gradually complex flow phenomena.

EN.535.742. Applied Machine Learning for Mechanical Engineers. 3 Credits.

This course covers machine learning fundamentals (e.g., optimization, perceptron, and universal approximation), some popular and advanced machine learning techniques (e.g., Supervised, Unsupervised, Probabilistic, Convolutional, and Generative Networks), and supercomputing techniques (with a focus on MARCC) to address mechanical engineering-related machine learning problems. The course requires Python 3+ programming skills; a free 3-hour Python 3+ tutorial will be provided to those who need to learn Python.

For current faculty and contact information go to http://www.mathematics.jhu.edu/people/