# APPLIED MATHEMATICS AND STATISTICS

http://engineering.jhu.edu/ams (http://engineering.jhu.edu/ams/)

The Department of Applied Mathematics and Statistics is devoted to the study and development of mathematical disciplines especially oriented to the complex problems of modern society. A broad undergraduate and graduate curriculum emphasizes several branches of applied mathematics: *Probability*, the mathematical representation and modeling of uncertainty; *Statistics*, the analysis and interpretation of data; *Operations Research*, the design, analysis, and improvement of actual operations and processes; *Optimization*, the determination of best or optimal decisions; *Discrete Mathematics*, the study of finite structures, arrangements, and relations; *Scientific Computation*, which includes all aspects of numerical computing in support of the sciences; and *Financial Mathematics*, the modeling and analysis of financial markets.

Probability and Statistics is treated in the curriculum as a single general area, dealing in a unified way with theory and methodology for probabilistic representation of chance phenomena, applications of stochastic modeling to physical and social sciences, formulation of statistical models, fitting of statistical models to data, and interpretation of data. Operations Research and Optimization represents a second general area, dealing in unified fashion with the application of optimization theory, mathematical programming, computer modeling, stochastic modeling, and game theory to planning and policy problems such as scheduling, allocation of resources, and facility location. Discrete Mathematics includes the traditional themes of graph theory and combinatorics, as well as newer topics arising from modern technological and theoretical developments. The fourth general area, Computational and Applied Mathematics, covers topics pertaining to computing, numerical analysis, advanced matrix analysis, and mathematical modeling. Financial Mathematics addresses applications by making use of applied mathematics techniques and models from many of the above-mentioned areas.

In its fundamental role of representing applied mathematics at Johns Hopkins University, the Department of Applied Mathematics and Statistics is complemented by the Department of Mathematics, with its differing emphasis. Located in the School of Engineering, the Department of Applied Mathematics and Statistics fulfills a special integrative role, stemming in part from the affinity of engineers for applied mathematics and in part from the increasing need for interaction between science and engineering. The mathematical sciences, especially the mathematics of modeling, provide a common language and tools through which engineers can develop closer alliances and cooperation with other scientists.

The department's degree programs include foundational and introductory course work drawing from all areas of the curriculum, along with specialized course work in areas such as probability, statistics, operations research, and optimization. Students, in consultation with their advisors, may develop challenging individual programs. The department emphasizes mathematical reasoning, mathematical modeling, abstraction from the particular, and innovative application all in a problem-oriented setting. The aim is to prepare graduates for professional careers in the mathematical sciences and related areas, in academic institutions as well as in governmental, industrial, and research organizations.

The undergraduate major in applied mathematics and statistics leads to the B.A. and B.S. degrees. The graduate program leads to the M.S.E. and Ph.D. degrees. In addition, under a combined bachelor's/master's program, exceptionally able undergraduates may be admitted early to simultaneous graduate work.

## **Facilities**

The department is located in the Wyman Park Building. Office space and liberal access to computing facilities are provided to resident graduate students. A Reading/Commons Room provides the opportunity for informal discussions among faculty and graduate students. The university's Milton S. Eisenhower Library maintains an excellent collection of literature in the mathematical sciences, including all of the important current journals.

## **Undergraduate Programs**

The undergraduate major in applied mathematics and statistics may serve as preparation for employment as an applied mathematician, for graduate study in applied mathematics or related areas, or as general quantitative training for a career in business, medicine, or other fields. An undergraduate major in applied mathematics and statistics takes an individually tailored program of courses within the department and in the Department of Mathematics (calculus, and perhaps further courses such as differential equations, analysis, complex variables, topology, and modern algebra) and electives in science and engineering. By suitable choice of electives, heavy concentration in a specific field of engineering is possible.

In order to develop a sound program suited to individual needs and interests, the student should consult regularly with the faculty advisor. Additional advisory information may be obtained from the department office.

With the advice and consent of the faculty advisor, each student constructs an individualized program meeting the departmental major requirements. A written copy of the program should be on file with the faculty advisor, with whom it can be revised and updated from time to time.

## **Graduate Programs**

A wide variety of advanced courses, seminars, and research opportunities is available in the Department of Applied Mathematics and Statistics. In addition to graduate programs in probability, statistics, operations research, optimization, discrete mathematics, scientific computation, and financial mathematics, advanced study is possible in interdisciplinary topics in cooperation with other departments, particularly the departments of Biostatistics, Computer Science, Economics, Environmental Health and Engineering, Health Services Administration, Mathematics, and Sociology. A graduate student in the Department of Applied Mathematics and Statistics may thus develop a program that suits their individual interests and objectives.

## Requirements for the Bachelor's/Master's Program

Highly motivated and exceptionally well-qualified undergraduates may apply for admission to the combined bachelor's/master's program in applied mathematics and statistics. Interested students should apply no later than the fall semester of their senior year. The requirements for this program consist of those for the bachelor's and master's programs. Further information is available on the department website.

## Admission

To be admitted to an advanced degree program in the department, an applicant must show that they have the basic intellectual capacity and has acquired the skills necessary to complete the program successfully within a reasonable period of time. A faculty committee evaluates each applicant's credentials; there are no rigid requirements.

Prospective applicants should submit transcripts of previous academic work, letters of recommendation from persons qualified to evaluate the applicant's academic performance and potential for graduate study, a statement of purpose describing anticipated professional goals, and Graduate Record Examination (GRE) scores (not required for PhD applicants). Foreign students must submit scores from the Test of English as a Foreign Language (TOEFL) or International English Language Testing System (IELTS).

Most applicants have undergraduate majors in quantitative fields such as mathematics, statistics, engineering, or a field in the physical sciences, but any major is permitted. Regardless of the major, completion of a program in undergraduate mathematics at least through advanced calculus and linear algebra is essential to begin the normal graduate program.

## **Programs**

- Applied Mathematics and Statistics, Bachelor of Arts (https://ecatalogue.jhu.edu/engineering/full-time-residential-programs/degreeprograms/applied-mathematics-statistics/applied-mathematicsstatistics-bachelors-degrees/)
- Applied Mathematics and Statistics, Bachelor of Science (https://ecatalogue.jhu.edu/engineering/full-time-residential-programs/degreeprograms/applied-mathematics-statistics/applied-mathematicsstatistics-bs/)
- Applied Mathematics and Statistics, Master of Science in Engineering (https://e-catalogue.jhu.edu/engineering/full-time-residentialprograms/degree-programs/applied-mathematics-statistics/appliedmathematics-statistics-master-science-engineering/)
- Applied Mathematics and Statistics, Minor (https://ecatalogue.jhu.edu/engineering/full-time-residential-programs/degreeprograms/applied-mathematics-statistics/applied-mathematicsstatistics-minor/)
- Applied Mathematics and Statistics, PhD (https://ecatalogue.jhu.edu/engineering/full-time-residential-programs/degreeprograms/applied-mathematics-statistics/applied-mathematicsstatistics-phd/)
- Data Science, Master's Degree (https://e-catalogue.jhu.edu/ engineering/full-time-residential-programs/degree-programs/appliedmathematics-statistics/data-science-masters-degree/)
- Financial Mathematics, Master of Science in Engineering (https://ecatalogue.jhu.edu/engineering/full-time-residential-programs/degreeprograms/applied-mathematics-statistics/financial-mathematicsmaster-science-engineering/)

For current course information and registration go to https://sis.jhu.edu/ classes/

## Courses

# EN.553.100. Introduction to Applied Mathematics and Statistics. 1 Credit.

A seminar-style series of lectures and assignments to acquaint the student with a range of intellectual and professional activities performed by applied mathematicians and statisticians. Problems arising in applied mathematics and statistics are presented by department faculty and outside speakers. Recommended Course Background: one semester of Calculus.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

# EN.553.101. Freshman Experience in Applied Mathematics & Statistics. 1 Credit.

The aim of this course is to provide students with an opportunity to work on a project in a small group setting together with an AMS faculty member. Projects can be varied in nature depending on the faculty member working with a group. The goal of a group could be to develop knowledge of a domain area in which mathematics is applied, to develop knowledge of some technique(s) in applied mathematics, to bring applied mathematics to bear on some application, or to develop knowledge in some foundational topic in mathematics. Faculty will present possible topics to students in the first week of classes. Students will be asked to rank their interests (first choice, second choice, etc.), and will provide their schedules. Based on their preferences, their schedules, and subject to group size limitations, students will be organized into groups of size at most 3, and will be assigned to course sections in the second week of classes. One faculty member will lead each section and will arrange to meet with the group once per week for an hour.

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.111. Statistical Analysis I. 4 Credits.

First semester of a general survey of statistical methodology. Topics include descriptive statistics, introductory probability, conditional probability, random variables, expectation, sampling, the central limit theorem, classical and robust estimation, confidence intervals, and hypothesis testing. Case studies from psychology, epidemiology, economics and other fields serve to illustrate the underlying theory. Some use of Minitab, Excel or R, but no prior computing experience is necessary. Recommended Course Background: four years of high school mathematics. Students who may wish to undertake more than two semesters of probability and statistics should consider EN.553.420-EN.553.430.

**Prerequisite(s):** Statistics Sequence restriction: students who have completed AS.230.205 may not enroll.;Statistics Sequence restriction: students who have completed any of these courses may not register. EN.553.211 OR EN.553.230 OR EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421 OR EN.553.430 OR EN.553.413 OR EN.560.435 OR AS.280.345 OR AS.200.314 OR AS.200.315 OR EN.560.348

## EN.553.112. Statistical Analysis II. 4 Credits.

Second semester of a general survey of statistical methodology. Topics include two-sample hypothesis tests, analysis of variance, linear regression, correlation, analysis of categorical data, and nonparametrics. Students who may wish to undertake more than two semesters of probability and statistics should strongly consider the EN.553.420-430 sequence.

Prerequisite(s): EN.553.111 OR AS.230.205 OR AS.280.345 OR credit for AP Statistics

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.122. Chance and Risk. 3 Credits.

The course is intended for humanities and social science majors. It will help students develop an appreciation of probability and randomness, and an understanding of its applications in real life situations involving chance and risk. Applications, controversies, and paradoxes involving risk in business and economics, health and medicine, law, politics, sports, and gambling will be used to illustrate probabilistic concepts such as independence, conditional probability, expectation, correlation, and variance. Class periods will typically include a combination of presentation of new material, an in-class activity, and class discussion. Attendance and class participation will be an important part of the learning experience. Prerequisites: There is no prerequisite beyond high school mathematics. The course is not open to students who have taken calculus.

**Prerequisite(s):** Students may not have completed AS.110.106 OR AS.110.107 OR AS.110.108 OR AS.110.109 OR AS.110.113 OR AS.110.202 OR AS.110.211

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.171. Discrete Mathematics. 4 Credits.

Introduction to the mathematics of finite systems. Logic; Boolean algebra; induction and recursion; sets, functions, relations, equivalence, and partially ordered sets; elementary combinatorics; modular arithmetic and the Euclidean algorithm; group theory; permutations and symmetry groups; graph theory. Selected applications. The concept of a proof and development of the ability to recognize and construct proofs are part of the course. Recommended Course Background: Four years of high school mathematics.

**Prerequisite(s):** Students may only earn credit for one of the following: EN.553.171, EN.553.172, OR EN.601.230.EN.553.171 may not be taken after EN.553.471 OR EN.553.472 OR EN.553.671 OR EN.553.672. **Corequisite(s):** EN.553.171 may not be taken concurrently with EN.553.471 or EN.553.472 or EN.553.671 or EN.553.672. Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.172. Honors Discrete Mathematics. 4 Credits.

Introduction to the mathematics of finite systems. Logic; Boolean algebra; induction and recursion; sets, functions, relations, equivalence, and partially ordered sets; elementary combinatorics; modular arithmetic and the Euclidean algorithm; polynomials rings, group theory; permutations groups and Galois theory; graph theory. Selected applications. The concept of a proof and development of the ability to recognize and construct proofs and analyze algorithms are part of the course. Recommended Course Background: Four years of high school mathematics.

**Prerequisite(s):** Students may only earn credit for one of the following: EN.553.171, EN.553.172, OR EN.601.230.EN.553.172 may not be taken after EN.553.471 OR EN.553.472 OR EN.553.671 OR EN.553.672. Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.211. Probability and Statistics for the Life Sciences. 4 Credits.

This is an introduction to statistics aimed at students in the life sciences. The course will provide the necessary background in probability with treatment of independence, Bayes theorem, discrete and continuous random variables and their distributions. The statistical topics covered will include sampling and sampling distributions, confidence intervals and hypothesis testing for means, comparison of populations, analysis of variance, linear regression and correlation. Analysis of data will be done using Excel.

**Prerequisite(s):** AS.110.106[C] OR AS.110.107[C] OR AS.110.108[C] OR AS.110.109[C] OR AS.110.113[C];Statistics Sequence restriction: Students who have completed any of these courses may not register. EN.550.230 OR AS.280.345 OR AS.200.314 OR AS.200.315 OR EN.550.310 OR EN.550.311 OR EN.560.435 OR EN.550.420 OR EN.550.430 OR EN.560.348

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.291. Linear Algebra and Differential Equations. 4 Credits.

An introduction to the basic concepts of linear algebra, matrix theory, and differential equations that are used widely in modern engineering and science. Intended for engineering and science majors whose program does not permit taking both AS.110.201 and AS.110.302. **Prerequisite(s):** AS.110.107 OR AS.110.109 OR AS.110.113 Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.295. Linear Algebra for Data Science. 4 Credits.

A thorough introduction to linear algebra, with a focus on applications to data science and statistics. Topics include linear algebra in Euclidean spaces: matrices, vectors, linear independence, determinants, subspaces, bases, change of coordinates, linear transformations, null spaces and ranges, projections, orthogonalization, eigenvalues and eigenvectors; as well as least-squares approximation, spectral decomposition, quadratic forms, convexity, principal component analysis, dimensionality reduction, and approximation in function spaces. Matlab will be used for computation and applications. Prerequisites: AS.110.107 OR AS.110.109 OR AS.110.113

**Prerequisite(s):** AS.110.107 OR AS.110.109 OR AS.110.113 Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

# EN.553.310. Probability & Statistics for the Physical Sciences & Engineering. 4 Credits.

An introduction to probability and statistics at the calculus level, intended for engineering and science students planning to take only one course on the topics. Combinatorial probability, independence, conditional probability, random variables, expectation and moments, limit theory, estimation, confidence intervals, hypothesis testing, tests of means and variances, goodness-of-fit. Recommended co-requisite: multivariable calculus.Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course.

**Prerequisite(s):** AS.110.109 OR AS.110.113;Statistics Sequence restriction: Students who have completed any of these courses may not register:EN.553.311 OR EN.560.435 OR EN.553.420 OR EN.553.421 OR EN.553.430 OR EN.560.348

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.311. Intermediate Probability and Statistics. 4 Credits.

This course will be at the same technical level as EN.553.310. Students are encouraged to consider EN.553.420-430 instead. Combinatorial probability, independence, conditional probability, random variables, expectation and moments, limit theory, estimation, confidence intervals, hypothesis testing, tests of means and variances, andgoodness-of-fit will be covered. Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course. Recommended Course Corequisite: AS.110.202

**Prerequisite(s):** AS.110.109 OR AS.110.113;Statistics Sequence restriction: students who have completed any of these courses may not register: EN.553.310 OR EN.560.435 OR EN.553.420 EN.553.421 OR EN.553.430 OR EN.560.348

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.312. Intermediate Probability and Statistics II. 3 Credits.

Continuation of Intermediate Probability and Statistics I. Junior-level treatment. Review of distribution theory, central limit theorem, normal random samples, independent and paired samples, hypothesis testing. Material includes Analysis of Variance, completely randomized designs, randomized block designs, factorial designs, multiple comparison methods, Boole-Bonferroni and Tukey methods. Non-parametric statistics, Wilcoxon rank sum, Wilcoxon signed rank, Sign test, Kruskal-Wallis test, Kolmogorov-Smirnov test, tests of randomness, difference sign test, turning point tests, runs test. Simple linear regression, least squares, correlation, coefficient of determination, estimation and prediction errors. Multiple linear regression, test of usefulness, higher-order models, evaluating models and fit. As time permits, Analysis of Covariance, generalized linear models, logistic regression, Poisson regression, nonlinear models.

**Prerequisite(s):** Students may not have completed EN.553.420, EN.553.421, EN.553.430, OR EN.553.431;EN.553.310 OR EN.553.311 Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.335. Mathematics for a Better World. 3 Credits.

This course offers and opportunity for students to work on projects from community organizations and nonprofits in Baltimore. Projects varies from year to year. Students will work in teams on the projects. Most projects involve data. Tasks may include data analysis, data visualization, data modeling and prediction. There is no prerequisite for this class. Students with quantitative and computing background, and passion for community engagement welcome.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.361. Introduction to Optimization I. 4 Credits.

An introductory survey of optimization methods, supporting mathematical theory and concepts, and application to problems of planning, design, prediction, estimation, and control in engineering, management, and science. Study of varied optimization techniques including linear programming, network-problem methods, dynamic programming, integer programming, and nonlinear programming. Students should be familiar with computing and linear algebra. Prerequisite: one year of calculus. Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course.

**Prerequisite(s):** (AS.110.107 OR AS.110.109 OR AS.110.113) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.362. Introduction to Optimization II. 4 Credits.

An introductory survey of optimization methods, supporting mathematical theory and concepts, and application to problems of planning, design, prediction, estimation, and control in engineering, management, and science. Study of varied optimization techniques including linear programming, network-problem methods, dynamic programming, integer programming, and nonlinear programming. **Prerequisite(s):** EN.550.361 AND (AS.110.202 OR AS.110.211 ) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.371. Cryptology and Coding. 4 Credits.

Computing experience. A first course in the mathematical theory of secure and reliable electronic communication. Cryptology is the study of secure communication: How can we ensure the privacy of messages? Coding theory studies how to make communication reliable: How can messages be sent over noisy lines? Topics include finite field arithmetic, error-detecting and error-correcting codes, data compressions, ciphers, one-time pads, the Enigma machine, one-way functions, discrete logarithm, primality testing, secret key exchange, public key cryptosystems, digital signatures, and key escrow.

Prerequisite(s): (EN.553.171 OR EN.553.172) AND (EN.553.2910R EN.553.295 OR AS.110.201 OR AS.110.212)

## EN.553.385. Introduction to Computational Mathematics. 4 Credits.

This course offers a broad introduction to the area of numerical computation in the mathematical sciences. Topics include but may not be limited to: floating point numbers, linear systems, LU factorization, vector and matrix norms, conditioning of linear systems, QR factorizations, the root finding problem, fixed point iterations, Newton's method in one variable and for nonlinear systems, interpolation, cubic splines, finite differences, numerical integration, initial-value problems for ODEs, Euler's method, systems of differential equations, Runge-Kutta methods, eigenvalue and singular value decomposition of matrices, polynomial approximation, shooting methods for boundary-value problems; time permitting: basic finite-difference methods for the solution of PDEs. The prerequisites are linear algebra, multivariable calculus, and differential equations; however, the latter may be taken concurrently. **Prerequisite(s):** (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212 ) AND (AS.110.202 OR AS.110.211 )

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.391. Dynamical Systems. 4 Credits.

Mathematical concepts and methods for describing and analyzing linear and nonlinear systems that evolve over time. Topics include boundedness, stability of fixed points and attractors, feedback, optimality, Liapounov functions, bifurcation, chaos, and catastrophes. Examples drawn from population growth, economic behavior, physical and engineering systems. The main mathematical tools are linear algebra and basic differential equations.

Prerequisite(s): EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.211

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.400. Mathematical Modeling and Consulting. 4 Credits.

Creating, analyzing and evaluating optimization and mathematical models using case studies. Project-oriented practice and guidance in modeling techniques, with emphasis on communication of methods and results. Applications may include transportation networks, scheduling, industrial processes, and telecommunications. Computation will be emphasized throughout using MATLAB.

Prerequisite(s): EN.553.361 OR EN.553.362;Students may receive credit for EN.550.400/EN.553.400 or EN.553.600, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.401. Introduction to Research in Applied Mathematics and Statistics. 3 Credits.

This course will provide students with a comprehensive hands-on introduction to the research process, which includes performing a literature survey, mathematical modeling, theoretical, numerical, and/or empirical approaches while solving some of the problems in the research landscape. Some topics include (but are unlimited to) the following: • Systems Reliability Analysis • Systems Management• Network Analysis• Sports Scheduling• 5-G (and Next-Gen) Explorations• Mechanical Ventilation Data Analysis and ManagementIn the process, students will have the experience to produce their status reports and results orally and in writing where the goal is to provide an initial working publication that can be extended for upcoming conference and/or journal submissions. **Prerequisite(s):** Students who have taken or are enrolled in EN.553.601 are not eligible to take EN.553.401.

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

# EN.553.402. Research and Design in Applied Mathematics: Data Mining. 4 Credits.

The course will be project oriented with focus on practical uses of machine learning and data mining. Throughout the semester, teams of 4 will work on topics decided by the students and the instructor. **Prerequisite(s):** EN.553.436

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

EN.553.404. Foundational Methods in Applied Mathematics. 4 Credits. Targeted overview of key mathematical techniques essential to advanced AMS courses. Basics of convergence and limits. Review of multivariable calculus, including differentiability of vector-valued functions; multiple integrals and applications; implicit and inverse function theorems; interchanges of limits; change of variables. Review of linear algebra, including matrix operations, factorizations, and spectral decompositions; determinants; special matrices. Constrained optimization. Note: multivariable calculus and linear algebra are prerequisites for this course. **Prerequisite(s):** Students who have taken or are enrolled in EN.553.604 are not eligible to take EN.553.404.;(AS.110.202 OR AS.110.211) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.413. Applied Statistics & Data Analysis I. 4 Credits.

An introduction to basic concepts, techniques, and major computer software packages in applied statistics and data analysis. Topics include numerical descriptive statistics, observations and variables, sampling distributions, statistical inference, linear regression, multiple regression, design of experiments, nonparametric methods, and sample surveys. Real-life data sets are used in lectures and computer assignments. Intensive use of statistical packages such as R to analyze data. **Prerequisite(s):** Students may receive credit for EN.550.413/EN.553.413 or EN.553.613, but not both.;EN.553.112 OR EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.414. Applied Statistics and Data Analysis II. 3 Credits.

Part II of a sequence on data analysis and linear models. Topics include categorical and discrete data analysis, mixed models, semiparametric and nonparametric regression, and generalized additive models.

Applications of these methods using the R environment for statistical computing will be emphasized.

Prerequisite(s): EN.550.413;Students may receive credit for EN.550.414/ EN.553.414 or EN.553.614, but not both.

## EN.553.420. Probability. 4 Credits.

Probability and its applications, at the calculus level. Emphasis on techniques of application and on rigorous mathematical demonstration. Probability, combinatorial probability, random variables, distribution functions, important probability distributions, independence, conditional probability, moments, covariance and correlation, limit theorems. Students initiating graduate work in probability or statistics should enroll in EN.553.620 or EN.553.720. Prerequisites: one year of calculus. Corequisites: multivariable calculus and linear algebra.Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course.

Prerequisite(s): Students may receive credit for one of the following: EN.550.420/EN.553.420 OR EN.553.421 OR EN.553.620.;AS.110.109 OR AS.110.113;AS.110.201 OR AS.110.202 OR AS.110.211 OR AS.110.212 OR EN.553.291 OR EN.553.295, can be taken concurrently.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.421. Honors Probability. 4 Credits.

Probability and its applications, at the calculus level. Emphasis on techniques of application and on rigorous mathematical demonstration. Probability, combinatorial probability, random variables, distribution functions, important probability distributions, independence, conditional probability, moments, exchangeability, joint distributions, conditional distributions and expectation, covariance and correlation, limit theorems. The honors version of this course will have enrichment exercises that explore and extend ideas learned in the ordinary lecture. Students initiating graduate work in probability or statistics should enroll in EN.550.620. Auditors are not permitted. Recommended Course Background: one year of calculus and mathematical maturity; Correquisite: multivariable calculus. By permission of the instructor or by recommendation of an AMS faculty member.

Prerequisite(s): AS.110.109 OR AS.110.113;AS.110.201 OR AS.110.202 OR AS.110.212 OR EN.553.291 OR EN.553.295, can be taken concurrently.

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.426. Introduction to Stochastic Processes. 4 Credits.

Mathematical theory of stochastic processes. Emphasis on deriving the dependence relations, statistical properties, and sample path behavior including random walks, Markov chains (both discrete and continuous time), Poisson processes, martingales, and Brownian motion. Applications that illuminate the theory. Students may receive credit for EN.553.426 or EN.553.626.

**Prerequisite(s):** Students may receive credit for EN.550.426/EN.553.426 or EN.553.626, but not both.;(EN.553.420 OR EN.553.421 OR EN.553.620 ) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212 ) **Corequisite(s):** Students may not enroll in EN.553.420 in the same semester.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.427. Stochastic Processes and Applications to Finance I. 4 Credits.

A development of stochastic processes with substantial emphasis on the processes, concepts, and methods useful in mathematical finance. Relevant concepts from probability theory, particularly conditional probability and conditional expection, will be briefly reviewed. Important concepts in stochastic processes will be introduced in the simpler setting of discrete-time processes, including random walks, Markov chains, and discrete-time martingales, then used to motivate more advanced material. Most of the course will concentrate on continuoustime stochastic processes, particularly martingales, Brownian motion, diffusions, and basic tools of stochastic calculus. Examples will focus on applications in finance, economics, business, and actuarial science.Students may only earn credit for one of EN.553.427 or EN.553.627.

**Prerequisite(s):** Students may receive credit for only one of EN.550.427, EN.553.427, OR EN.553.627;EN.553.420 OR EN.553.421 OR EN.553.620 Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.430. Mathematical Statistics. 4 Credits.

Introduction to mathematical statistics. Finite population sampling, approximation methods, classical parametric estimation, hypothesis testing, analysis of variance, and regression. Bayesian methods. **Prerequisite(s):** Students may receive credit for EN.550.430/EN.553.430 or EN.553.630 or EN.553.431, but not all.;(EN.553.420 OR EN.553.421 OR EN.553.620) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295), can be taken concurrently.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.431. Honors Mathematical Statistics. 4 Credits.

Introduction to the theory and methodology of mathematical statistics: parametric estimation, including asymptotic properties of estimators and approximation methods; hypothesis testing; analysis of variance; regression; bootstrapping and nonparametrics. Intended for students with a particular interest in the theoretical foundations of statistical procedures.

**Prerequisite(s):** Students may receive credit for only one of EN.553.430, EN.553.431 or EN.553.630.;(EN.553.420 OR EN.553.421 OR EN.553.620) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295) AND (AS.110.202 OR AS.110.211)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.432. Bayesian Statistics. 3 Credits.

The course will cover Bayesian methods for exploratory data analysis. The emphasis will be on applied data analysis in various disciplines. We will consider a variety of topics, including introduction to Bayesian inference, prior and posterior distribution, hierarchical models, spatial models, longitudinal models, models for categorical data and missing data, model checking and selection, computational methods by Markov Chain Monte Carlo using R or Matlab. We will also cover some nonparametric Bayesian models if time allows, such as Gaussian processes and Dirichlet processes.

**Prerequisite(s):** Students may take only one of EN.550.632, EN.553.432, EN.553.632 or EN.553.732.;(EN.553.420 OR EN.553.421 OR EN.553.620) AND (EN.553.430 OR EN.553.431 OR EN.553.630)

#### EN.553.433. Monte Carlo Methods. 4 Credits.

The objective of the course is to survey essential simulation techniques for popular stochastic models. The stochastic models may include classical time-series models, Markov chains and diffusion models. The basic simulation techniques covered will be useful in samplegeneration of random variables, vectors and stochastic processes, and as advanced techniques, importance sampling, particle filtering and Bayesian computation may be discussed.

**Prerequisite(s):** Students may receive credit for EN.553.433 or EN.553.633, but not both.;EN.553.430 OR EN.553.431 OR EN.553.630 Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.434. Elements of Statistical Learning. 4 Credits.

Rigorous mathematical analysis of statistical learning models, with an emphasis on underlying theory, along with integrated computation and applications. Brief review of background and an introduction to learning problems, followed by regression (linear, logistic, lasso, and kernel), support vector machines, clustering, principal component analysis, LDA, EM, and deep learning. Covers computational aspects, including optimization, approximation-generalization and bias-variance tradeoffs, model validation, and selection.

**Prerequisite(s):** EN.553.430 OR EN.553.431 OR EN.553.630 Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.436. Introduction to Data Science. 4 Credits.

Today the term Data Science is widely used covering a broad range of topics from mathematics and algorithms to actual data analysis and machine learning techniques. This course provides a thorough survey of relevant methods balancing the theory and the application aspects. Accordingly, the material and the discussions alternate between the methodology along with its underlying assumptions and the implementations along with their applications. We will cover several supervised methods for regression and classification, as well as unsupervised methods for clustering and dimensional reduction. To name a few in chronological order, the topics will include generalized linear regression, principal component analysis, nearest neighbor and Bayesian classifiers, support vector machines, logistic regression, decision trees, random forests, K-means clustering, Gaussian mixtures and Laplacian eigenmaps. The course uses Python and Jupyter Notebook and includes visualization techniques throughout the semester. Time permitting, an introduction to the Structured Query Language (SQL) is provided toward the end of the semester.

**Prerequisite(s):** Students may receive credit for EN.550.436/EN.553.436 or EN.553.636, but not both.;(AS.110.202 OR AS.110.211) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.439. Time Series Analysis. 3 Credits.

Time series analysis from the frequency and time domain approaches. Descriptive techniques; regression analysis; trends, smoothing, prediction; linear systems; serial correlation; stationary processes; spectral analysis.

**Prerequisite(s):** Students may receive credit for EN.553.439 or EN.553.639, but not both.;(EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421 OR EN.553.620) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.441. Equity Markets and Quantitative Trading. 3 Credits.

This course introduces equity markets from a mathematical point of view. The properties of equities and equity-linked instruments will be described. Several quantitative trading strategies will be studied. Order execution tactics and the effect of market structure will be analyzed. Students will select a specialized aspect of the equity markets to investigate and complete a related independent project.

**Prerequisite(s):** Students may receive credit for EN.550.441/EN.553.441 or EN.553.641, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.442. Investment Science. 4 Credits.

This course offers a rigorous treatment of the subject of investment as a scientific discipline. Mathematics is employed as the main tool to convey the principles of investment science and their use to make investment calculations for good decision-making. Topics covered in the course include the basic theory of interest and its application to fixed-income securities, cash flow analysis and capital budgeting, mean-variance portfolio theory, and the associated capital asset pricing model, utility function theory and risk analysis, derivative securities and basic option theory, portfolio evaluation. The student is expected to be comfortable with the use of mathematics as a method of deduction and problem solving. Students may not receive credit for both EN.550.342 and EN.553.442Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course.

**Prerequisite(s):** Students may receive credit for only one of EN.550.342, EN.550.442, EN.553.442 or EN.553.642;(AS.110.109 OR AS.110.113) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212) AND (EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421 OR EN.553.620 OR EN.553.430 OR EN.553.431 OR EN.553.630) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.444. Introduction to Financial Derivatives. 4 Credits.

This course will develop the mathematical concepts and techniques for modeling cash instruments and their hybrids and derivatives. **Prerequisite(s):** Students may receive credit for EN.550.444/EN.553.444 or EN.553.644, but not both.;AS.110.302 AND (EN.553.420 OR EN.553.421)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.445. Interest Rate and Credit Derivatives. 4 Credits.

Advances in corporate finance, investment practice and the capital markets have been driven by the development of a mathematically rigorous theory for financial instruments and the markets in which they trade. This course builds on the concepts, techniques, instruments and markets introduced in EN.553.444. In addition to new topics in credit enhancement and structured securities, the focus is expanded to include applications in portfolio theory and risk management, and covers some numerical and computational approaches.

**Prerequisite(s):** EN.553.444 OR EN.553.644;Students may receive credit for EN.550.445/EN.553.445 or EN.553.645, but not both.

## EN.553.446. Risk Measurement/Management in Financial Markets. 4 Credits.

This course applies advanced mathematical techniques to the measurement, analysis, and management of risk. The focus is on financial risk. Sources of risk for financial instruments (e.g., market risk, interest rate risk, credit risk) are analyzed; models for these risk factors are studied and the limitation, shortcomings and compensatory techniques are addressed.

Prerequisite(s): Students may receive credit for EN.550.446/EN.553.446 or EN.553.646, but not both.;EN.553.444 OR EN.553.644

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.447. Quantitative Portfolio Theory and Performance Analysis. 4 Credits.

This course focuses on modern quantitative portfolio theory, models, and analysis. Topics include intertemporal approaches to modeling and optimizing asset selection and asset allocation; benchmarks (indexes), performance assessment (including, Sharpe, Treynor and Jenson ratios) and performance attribution; immunization theorems; alpha-beta separation in management, performance measurement and attribution; Replicating Benchmark Index (RBI) strategies using cash securities / derivatives; Liability-Driven Investment (LDI); and the taxonomy and techniques of strategies for traditional management: Passive, Quasi-Passive (Indexing) Semi-Active (Immunization & Dedicated) Active (Scenario, Relative Value, Total Return and Optimization). In addition, risk management and hedging techniques are also addressed. **Prerequisite(s)**: Students may receive credit for 550.447/553.447 OR

EN.553.647, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.449. Advanced Equity Derivatives. 4 Credits.

This course will cover the pricing, trading and risk management of equity derivatives, with emphasis on more exotic derivatives such as path-dependent and multi-asset derivatives. The course will emphasize practical issues: students will build their own pricing and risk management tools, and gain experience simulating the dynamic hedging of a complex derivatives portfolio. Students will practice structuring and selling equity derivative products. Pricing issues such a model selection, unobservable input parameters and calibration will be discussed, and students will learn techniques to manage the often highly nonlinear and discontinuous risks associated with these products. The course will have a significant computing component: both in the classroom and as homework projects, students will use Excel, write VBA macros and write and call C++ routines in the Microsoft Windows environment (which is the most common computing environment used by the financial industry). Prerequisite(s): Students may receive credit for EN.550.449/EN.553.449 or EN.553.649, but not both.; EN.553.444 OR EN.553.644

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.450. Computational Molecular Medicine. 4 Credits.

Computational systems biology has emerged as the dominant framework for analyzing high-dimensional "omics" data in order to uncover the relationships among molecules, networks and disease. In particular, many of the core methodologies are based on statistical modeling, including machine learning, stochastic processes and statistical inference. We will cover the key aspects of this methodology, including measuring associations, testing multiple hypotheses, and learning predictors, Markov chains and graphical models. In addition, by studying recent important articles in cancer systems biology, we will illustrate how this approach enhances our ability to annotate genomes, discover molecular disease networks, detect disease, predict clinical outcomes, and characterize disease progression. Whereas a good foundation in probability and statistics is necessary, no prior exposure to molecular biology is required (although helpful).

**Prerequisite(s):** Students may receive credit for EN.553.450 or EN.553.650, but not both.;(EN.553.420 OR EN.553.421 OR EN.553.620) AND (EN.553.430 OR EN.553.431 OR EN.553.630) OR equivalent courses in probability and statistics.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.453. Mathematical Game Theory. 4 Credits.

Mathematical analysis of cooperative and noncooperative games. Theory and solution methods for matrix game (two players, zero-sum payoffs, finite strategy sets), games with a continuum of strategies, N-player games, games in rule-defined form. The roles of information and memory. Selected applications to economic, recreational, and military situations. Prereq: Multivariable Calculus, probability, linear algebra.

**Prerequisite(s):** Students may receive credit for EN.550.453/EN.553.453 or EN.553.653, but not both.;(AS.110.202 OR AS.110.211) AND (EN.553.420 OR EN.553.620) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.463. Network Models in Operations Research. 4 Credits.

In-depth mathematical study of network flow models in operations research, with emphasis on combinatorial approaches for solving them. Introduction to techniques for constructing efficient algorithms, and to some related data structures, used in solving shortest-path, maximumvolume, flow, and minimum-cost flow problems. Emphasis on linear models and flows, with brief discussion of non-linear models and network design.

**Prerequisite(s):** Students may receive credit for EN.550.463/EN.553.463 or EN.553.663, but not both.;EN.553.361 OR EN.553.661 OR EN.553.761 OR EN.553.461

#### EN.553.465. Introduction to Convexity. 4 Credits.

Convexity is a simple mathematical concept that has become central in a diverse range of applications in engineering, science and business applications. Our main focus from the applications perspective will be the use of convexity within optimization problems, where convexity plays a key role in identifying the "easy" problems from the "hard" ones. The course will have an equal emphasis on expositing the rich mathematical structure of the field itself (properties of convex sets, convex functions, Helly-Caratheorody-Radon type theorems, polarity/duality, subdifferential calculus, polyhedral theory), and demonstrating how these ideas can be leveraged to model and solve optimization problems (via a detailed study of linear programming and basics of nonlinear convex optimization). Recommend Course Background: Familiarity with basic real analysis, linear algebra.

**Prerequisite(s):** Students may receive credit for EN.550.465/EN.553.465 or EN.553.665, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.471. Combinatorial Analysis. 4 Credits.

Counting techniques: generating functions, recurrence relations, Polya's theorem. Combinatorial designs: Latin squares, finite geometries, balanced incomplete block designs. Emphasis on problem solving. Recommended Course Background: AS.553.291 or AS.110.201Students who have received credit for AS.110.106 and/or AS.110.107 taken prior to Fall 2020 should contact the course instructor to determine whether they can receive permission to register for this course.

Prerequisite(s): Students may receive credit for EN.550.471/EN.553.471 or EN.550.671/EN.553.671, but not both.;(AS.110.109 OR AS.110.113) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295) **Corequisite(s):** EN.553.171 may not be taken concurrently with EN.553.471, EN.553.472, EN.553.671, or EN.553.672. Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.553.472. Graph Theory. 4 Credits.

Study of systems of "vertices" with some pairs joined by "edges." Theory of adjacency, connectivity, traversability, feedback, and other concepts underlying properties important in engineering and the sciences. Topics include paths, cycles, and trees; routing problems associated with Euler and Hamilton; design of graphs realizing specified incidence conditions and other constraints. Attention directed toward problem solving, algorithms, and applications. One or more topics taken up in greater depth.

**Prerequisite(s):** Students may receive credit for EN.550.472/EN.553.472 or EN.553.672, but not both.;EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212

Corequisite(s): EN.550.171 may not be taken concurrently with EN.550.471 or EN.550.472

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.480. Numerical Linear Algebra. 4 Credits.

A course on computational linear algebra and applications. Topics include floating-point arithmetic algorithms and convergence Gaussian elimination for linear systems matrix decompositions (LU, Cholesky, QR) iterative methods for systems (Jacobi, Gauss Seidel) approximation of eigenvalues (power method, QR-algorithm) and also singular values and singular-value decomposition (SVD). Theoretical topics such as vector spaces, inner products norms, linear operators, matrix norms, eigenvalues and canonical forms of matrices (Jordan, Schur) are reviewed as needed. Matlab is used to solve all numerical exercises.Students who took EN.553.385 prior to Fall 2023 may not take EN.553.480.

**Prerequisite(s):** Students who took EN.553.385 prior to Fall 2023 may not take EN.553.480.;(EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.481. Numerical Analysis. 4 Credits.

Brief review of topics in elementary numerical analysis such as floatingpoint arithmetic, Gaussian elimination for linear equations, interpolation and approximation. Core topics to be covered: numerical linear algebra including eigenvalue and linear least-squares problems, iterative algorithms for nonlinear equations and least squares problems, and convergence theory of numerical methods. Other possible topics: sparse matrix computations, numerical solution of partial differential equations, finite element methods, and parallel algorithms.

Prerequisite(s): Students may take only one of EN.550.681, EN.553.481, EN.553.681 or EN.553.781.;(AS.110.202 OR AS.110.211) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212) AND (EN.553.291 OR AS.110.302 OR AS.110.417 OR EN.553.386 OR EN.553.388 OR EN.553.391)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.483. Numerical Methods for Partial Differential Equations. 4 Credits.

We discuss numerical methods for solving partial differential equations, explaining how solution methods must be appropriate to the mathematical structure of the equation. Specific topics will be Hyperbolic PDE's (CFL stability condition, characteristics, convergence, nonlinear conservation laws, shock capturing), Parabolic PDE's (boundary conditions, explicit and implicit discretizations, consistency, stability, and convergence, operator splitting), Elliptic PDE's (iterative methods, variational formulations). We shall focus mainly on finite-difference schemes, but other methods such as finite element, finite volume, spectral methods, Chebyshev polynomials, etc. may be discussed as time permits. All numerical methods will be illustrated with Matlab scripts. Prior knowledge of Numerical Linear Algebra is strongly recommended. A good introductory course in the mathematics of Partial Differential Equations would also be helpful, but this material will be reviewed as necessary.

#### Prerequisite(s): EN.553.385 OR EN.553.480

## EN.553.488. Computing for Applied Mathematics. 3 Credits.

The aim of this course is to develop students' programming skills for solving problems commonly encountered in applied mathematics contexts. Specific problems that arise in applications of mathematics and data science (e.g. from finance, data analysis, or the physical sciences) are used to motivate concepts, techniques, and paradigms related to computation and programming. The Python language as well as a large collection of packages will be introduced. Students should be comfortable using computers but no prior programming background is required.

**Prerequisite(s):** Students may receive credit for EN.550.488/EN.553.488 or EN.553.688, but not both.;EN.553.310 OR EN.553.311 OR (EN.553.420 OR EN.553.421 AND (EN.553.430 OR EN.553.431))

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.491. Dynamical Systems. 4 Credits.

Mathematical concepts and methods for describing and analyzing linear and nonlinear systems that evolve over time. Topics include boundedness, stability of fixed points and attractors, feedback, optimality, Liapounov functions, bifurcation, chaos, and catastrophes. Examples drawn from population growth, economic behavior, physical and engineering systems. The main mathematical tools are linear algebra and basic differential equations.

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.492. Mathematical Biology. 3 Credits.

This course will examine the mathematical methods relevant to modeling biological phenomena, particularly dynamical systems and probability. Topics include ordinary differential equations and their simulation; stability and phase plane analysis; branching processes; Markov chains; and stochastically perturbed systems. Biological applications will be drawn from population growth, predator-prey dynamics, epidemiology, genetics, intracellular transport, and neuroscience.

**Prerequisite(s):** Students may receive credit for EN.553.492 or EN.553.692, but not both.;(EN.553.420 OR EN.553.421 OR EN.553.620) AND (AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295) AND (AS.110.302 OR AS.110.306 OR EN.553.291)

Distribution Area: Natural Sciences, Quantitative and Mathematical Sciences

AS Foundational Abilities: Science and Data (FA2)

## EN.553.493. Mathematical Image Analysis. 4 Credits.

This course gives an overview of various mathematical methods related to several problems encountered in image processing and analysis, and presents numerical schemes to address them. It will focus on problems like image denoising and deblurring, contrast enhancement, segmentation and registration. The different mathematical concepts shall be introduced during the course; they include in particular functional spaces such as Sobolev and BV, Fourier and wavelet transforms, as well as some notions from convex optimization and numerical analysis. Most of such methods will be illustrated with algorithms and simulations on discrete images, using MATLAB. Prerequisites : linear algebra, multivariate calculus, basic programming in MATLAB. Recommended Course Background: Real analysis

**Prerequisite(s):** Students may receive credit for EN.550.493/EN.553.493 or EN.553.693, but not both.;(AS.110.202 OR AS.110.211) AND (EN.553.291 OR EN.553.295 OR AS.110.201 OR AS.110.212) Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.494. Applied and Computational Multilinear Algebra. 3 Credits.

In this seminar we plan to discuss generalizations of theorems and algorithms from matrix theory to hypermatrices. More specifically the seminar will discuss hypermatrix/tensor algebras, rank, spectra and transforms. Using the python friendly free open-source mathematics software SageMath and the hypermatrix algebra package we will discuss applications of hypermatrices to combinatorics, machine learning and data analysis.Preliminary knowledge of the Python language is not required.

Prerequisite(s): Students may receive credit for EN.550.494/EN.553.494 or EN.553.694, but not both.;AS.110.201 OR AS.110.212 OR EN.553.291 OR EN.553.295

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

### EN.553.500. Undergraduate Research. 1 - 3 Credits.

Reading, research, or project work for undergraduate students. Prearranged individually between students and faculty.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

#### EN.553.501. Senior Thesis. 3 Credits.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

#### EN.553.502. Undergraduate Independent Study. 1 - 3 Credits.

Reading, research, or project work for undergraduate students. Prearranged individually between students and faculty. Recent topics and activities: percolation models, data analysis, course development assistance, and dynamical systems.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

## EN.553.506. Capstone Experience in Data Science. 3 - 6 Credits.

Project work for Data Science Master's students. Arranged individually between students and faculty.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

Distribution Area: Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

## EN.553.512. Group Undergraduate Research. 1 - 3 Credits.

Reading, research, or project work for undergraduate students. Prearranged meetings between students and faculty. This section has a weekly research group meeting that students are expected to attend. **Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

**EN.553.513.** Directed Reading in Applied Mathematics. 1 - 2 Credits. A small number of undergraduates are individually paired with PhD students. Each undergraduate will read by themself pages from a book and meet with their PhD mentor once a week for an hour, throughout the semester.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

### EN.553.552. Undergraduate Internship. 1 Credit.

**Prerequisite(s):** You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration, Online Forms.

## EN.553.600. Mathematical Modeling and Consulting. 4 Credits.

Creating, analyzing and evaluating optimization and mathematical models using case studies. Project-oriented practice and guidance in modeling techniques, with emphasis on communication of methods and results. Applications may include transportation networks, scheduling, industrial processes, and telecommunications. Computation will be emphasized throughout using MATLAB. Recommend Course Background: EN.553.361 OR EN.553.362.

**Prerequisite(s):** Students may receive credit for EN.550.400/EN.553.400 or EN.553.600, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.601. Introduction to Research in Applied Mathematics and Statistics. 3 Credits.

This course will provide students with a comprehensive hands-on introduction to the research process, which includes performing a literature survey, mathematical modeling, theoretical, numerical, and/or empirical approaches while solving some of the problems in the research landscape. Some topics include (but are unlimited to) the following: • Systems Reliability Analysis • Systems Management• Network Analysis• Sports Scheduling• 5-G (and Next-Gen) Explorations• Mechanical Ventilation Data Analysis and ManagementIn the process, students will have the experience to produce their status reports and results orally and in writing where the goal is to provide an initial working publication that can be extended for upcoming conference and/or journal submissions. **Prerequisite(s):** Students who have taken or are enrolled in EN.553.401 are not eligible to take EN.553.601.

Distribution Area: Quantitative and Mathematical Sciences

## EN.553.602. Research and Design in Applied Mathematics: Data Mining. 4 Credits.

The course will be project oriented with focus on practical uses of machine learning and data mining. Throughout the semester, teams of 4 will work on topics decided by the students and the instructor. **Prerequisite(s):** EN.553.636

Distribution Area: Quantitative and Mathematical Sciences

## **EN.553.604.** Foundational Methods in Applied Mathematics. 4 Credits. Targeted overview of key mathematical techniques essential to advanced AMS courses. Basics of convergence and limits. Review of multivariable calculus, including differentiability of vector-valued functions; multiple integrals and applications; implicit and inverse function theorems; interchanges of limits; change of variables. Review of linear algebra, including matrix operations, factorizations, and spectral decompositions; determinants; special matrices. Constrained optimization. Note: multivariable calculus and linear algebra are prerequisites for this course. **Prerequisite(s):** Students who have taken or are enrolled in EN.553.404 are not eligible to enroll in EN.553.604

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.613. Applied Statistics & Data Analysis I. 4 Credits.

An introduction to basic concepts, techniques, and major computer software packages in applied statistics and data analysis. Topics include numerical descriptive statistics, observations and variables, sampling distributions, statistical inference, linear regression, multiple regression, design of experiments, nonparametric methods, and sample surveys. Real-life data sets are used in lectures and computer assignments. Intensive use of statistical packages such as R to analyze data. Recommended Course Background: EN.553.112 or EN.553.310 or EN.553.311 or EN.553.420.

**Prerequisite(s):** Students may receive credit for EN.550.413/EN.553.413 or EN.553.613, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.614. Applied Statistics and Data Analysis II. 3 Credits.

Part II of a sequence on data analysis and linear models. Topics include categorical and discrete data analysis, mixed models, semiparametric and nonparametric regression, and generalized additive models. Applications of these methods using the R environment for statistical computing will be emphasized.

**Prerequisite(s):** Students may receive credit for EN.550.414/EN.553.414 or EN.553.614, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.620. Probability. 4 Credits.

Probability and its applications, at the calculus level. Emphasis on techniques of application and on rigorous mathematical demonstration. Probability, combinatorial probability, random variables, distribution functions, important probability distributions, independence, conditional probability, moments, covariance and correlation, limit theorems. Recommended course background: (AS.110.109 or AS.110.113) and previously or concurrently (AS.110.202 or AS.110.201 or AS.110.212). **Prerequisite(s):** Students may receive credit for only one of the following: EN.553.420 OR EN.553.421 OR EN.553.620.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.626. Introduction to Stochastic Processes. 4 Credits.

Mathematical theory of stochastic processes. Emphasis on deriving the dependence relations, statistical properties, and sample path behavior including random walks, Markov chains (both discrete and continuous time), Poisson processes, martingales, and Brownian motion. Applications that illuminate the theory. Students may receive credit for EN.553.426 or EN.553.626. Recommended course background: (EN.553.291 OR AS.110.201 OR AS.110.212).

Prerequisite(s): EN.553.620; Students may receive credit for EN.550.426/ EN.553.426 or EN.553.626, but not both.

**Corequisite(s):** Students may not enroll in EN.553.620 in the same semester.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.627. Stochastic Processes and Applications to Finance I. 4 Credits.

A development of stochastic processes with substantial emphasis on the processes, concepts, and methods useful in mathematical finance. Relevant concepts from probability theory, particularly conditional probability and conditional expection, will be briefly reviewed. Important concepts in stochastic processes will be introduced in the simpler setting of discrete-time processes, including random walks, Markov chains, and discrete-time martingales, then used to motivate more advanced material. Most of the course will concentrate on continuoustime stochastic processes, particularly martingales, Brownian motion, diffusions, and basic tools of stochastic calculus. Examples will focus on applications in finance, economics, business, and actuarial science. Recommend Course Background: Probability.

**Prerequisite(s):** Students may receive credit for only one of EN.550.427, EN.553.427, EN.553.627

Distribution Area: Quantitative and Mathematical Sciences

# EN.553.628. Stochastic Processes and Applications to Finance II. 4 Credits.

A basic knowledge of stochastic calculus and Brownian motion is assumed. Topics include stochastic differential equations, the Feynman-Kac formula and connections to partial differential equations, changes of measure, fundamental theorems of asset pricing, martingale representations, first passage times and pricing of path-dependent options, and jump processes.Student must pass EN.553.627 with a grade of B- or better to enroll in EN.553.628.

**Prerequisite(s):** Students may not have previously completed, be currently enrolled in, nor register concurrently in EN.553.620.;Students may receive credit for EN.550.428/EN.553.428 or EN.553.628, but not both.

Distribution Area: Quantitative and Mathematical Sciences

### EN.553.630. Mathematical Statistics. 4 Credits.

Introduction to the basic principles of mathematical statistics and data analysis. Emphasis on techniques of application. Classical parametric estimation, hypothesis testing, and multiple decision problems; linear models, analysis of variance, and regression; nonparametric and robust procedures; decision-theoretic setting, Bayesian methods. Recommended Course Background: EN.553.620 AND (AS.110.201 OR AS.110.212 OR EN.553.291 ).

**Prerequisite(s):** Students may receive credit for EN.550.430/EN.553.430 or EN.553.630, but not both.;Students must have completed EN.553.420 OR EN.553.421 OR EN.553.620 prior to enrolling in EN.553.630. Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.632. Bayesian Statistics. 3 Credits.

The course will cover Bayesian methods for exploratory data analysis. The emphasis will be on applied data analysis in various disciplines. We will consider a variety of topics, including introduction to Bayesian inference, prior and posterior distribution, hierarchical models, spatial models, longitudinal models, models for categorical data and missing data, model checking and selection, computational methods by Markov Chain Monte Carlo using R or Matlab. We will also cover some nonparametric Bayesian models if time allows, such as Gaussian processes and Dirichlet processes. Recommended prerequisites: EN.553.620 and (EN.553.630 or EN.553.730)

Prerequisite(s): Students may take only one of EN.550.632, EN.553.432, EN.553.632 or EN.553.732.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.633. Monte Carlo Methods. 4 Credits.

The objective of the course is to survey essential simulation techniques for popular stochastic models. The stochastic models may include classical time-series models, Markov chains and diffusion models. The basic simulation techniques covered will be useful in samplegeneration of random variables, vectors and stochastic processes, and as advanced techniques, importance sampling, particle filtering and Bayesian computation may be discussed. Recommended Course Background: EN.553.630.

**Prerequisite(s):** Students may receive credit for EN.550.433/EN.553.433 or EN.553.633, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.634. Elements of Statistical Learning. 4 Credits.

Rigorous mathematical analysis of statistical learning models, with an emphasis on underlying theory, along with integrated computation and applications. Brief review of background and an introduction to learning problems, followed by regression (linear, logistic, lasso, and kernel), support vector machines, clustering, principal component analysis, LDA, EM, and deep learning. Covers computational aspects, including optimization, approximation-generalization and bias-variance tradeoffs, model validation, and selection.

Prerequisite(s): EN.553.430 OR EN.553.431 OR EN.553.630 Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.636. Introduction to Data Science. 4 Credits.

Today the term Data Science is widely used covering a broad range of topics from mathematics and algorithms to actual data analysis and machine learning techniques. This course provides a thorough survey of relevant methods balancing the theory and the application aspects. Accordingly, the material and the discussions alternate between the methodology along with its underlying assumptions and the implementations along with their applications. We will cover several supervised methods for regression and classification, as well as unsupervised methods for clustering and dimensional reduction. To name a few in chronological order, the topics will include generalized linear regression, principal component analysis, nearest neighbor and Bayesian classifiers, support vector machines, logistic regression, decision trees, random forests, K-means clustering, Gaussian mixtures and Laplacian eigenmaps. The course uses Python and Jupyter Notebook and includes visualization techniques throughout the semester. Time permitting, an introduction to the Structured Query Language (SQL) is provided toward the end of the semester.

**Prerequisite(s):** Students may receive credit for EN.550.436/EN.553.436 or EN.553.636, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.639. Time Series Analysis. 3 Credits.

Time series analysis from the frequency and time domain approaches. Descriptive techniques; regression analysis; trends, smoothing, prediction; linear systems; serial correlation; stationary processes; spectral analysis. Recommended course background: EN.553.620 and (AS.110.201 OR AS.110.212 OR EN.553.291)

**Prerequisite(s):** Students may receive credit for EN.550.439/EN.553.439 or EN.553.639, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.640. Machine Learning in Finance. 3 Credits.

This course aims at helping students learn about how machine learning techniques are increasingly embraced by the field of finance. We will explore: (a) various topics and problems in finance that have been/will be benefited from the advances in machine learning, including but may not limited to portfolio optimization, asset pricing, market microstructure, high frequency trading, et cetera.; (b) different models of deep learning, (inverse) reinforcement learning and transfer learning that have been applied to tackle financial problems or have great potential in doing so. Recent advances, such as market prediction via special designs of neural networks, market simulator using generative adversarial networks, trading with reinforcement learning, portfolio optimization assisted by transfer learning and so on, will be discussed in this course. While this course is not intended to be highly theoretical, some familiarity with real analysis, optimization, probability and stochastic processes (Brownian motion, Markov processes, Poisson processes, martingales), and machine-learning would be helpful.

Prerequisite(s): (EN.553.427 OR EN.553.627) AND (EN.553.444 OR EN.553.644)

## EN.553.641. Equity Markets and Quantitative Trading. 3 Credits.

This course introduces equity markets from a mathematical point of view. The properties of equities and equity-linked instruments will be described. Several quantitative trading strategies will be studied. Order execution tactics and the effect of market structure will be analyzed. Students will select a specialized aspect of the equity markets to investigate and complete a related independent project.

**Prerequisite(s):** Students may receive credit for EN.550.441/EN.553.441 or EN.553.641, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.642. Investment Science. 4 Credits.

This course offers a rigorous treatment of the subject of investment as a scientific discipline. Mathematics is employed as the main tool to convey the principles of investment science and their use to make investment calculations for good decision-making. Topics covered in the course include the basic theory of interest and its application to fixed-income securities, cash flow analysis and capital budgeting, mean-variance portfolio theory, and the associated capital asset pricing model, utility function theory and risk analysis, derivative securities and basic option theory, portfolio evaluation. The student is expected to be comfortable with the use of mathematics as a method of deduction and problem solving. Recommended Course Background: (AS.110.109 OR AS.110.113 ) AND (EN.553.291 OR AS.110.201 OR AS.110.212 ) AND (EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.430). Prerequisite(s): Students may receive credit for EN.550.342 or EN.550.442/EN.553.442 or EN.553.642, but not both. Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.643. Energy Markets and Risk Management. 3 Credits.

The course objectives are to provide a deep understanding of commodities markets, with a focus on Energy (Natural Gas, Electricity, Renewable Energy, Crude Oil) and extension to clean energy and corresponding carbon emission markets. The important instruments (Forward, Futures, Options) will be redefined, valued and used in real risk management examples. This course provides an opportunity to bridge the gap between financial models in academy and risk management solutions in complicated energy markets. Students should have a background in probability and financial derivatives. Distribution Area: Quantitative and Mathematical Sciences

EN.553.644. Introduction to Financial Derivatives. 4 Credits.

## This course will develop the mathematical concepts and techniques for modeling cash instruments and their hybrids and derivatives. Prerequisites: background in Probability and Financial Derivatives. **Prerequisite(s):** Students may receive credit for EN.550.444/ EN.553.444 or EN.553.644, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.645. Interest Rate and Credit Derivatives. 4 Credits.

Advances in corporate finance, investment practice and the capital markets have been driven by the development of a mathematically rigorous theory for financial instruments and the markets in which they trade. This course builds on the concepts, techniques, instruments and markets introduced in EN.553.644. In addition to new topics in credit enhancement and structured securities, the focus is expanded to include applications in portfolio theory and risk management, and covers some numerical and computational approaches. Recommended Course Background: EN.553.644

**Prerequisite(s):** Students may receive credit for EN.550.445/EN.553.445 or EN.553.645, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

# EN.553.646. Risk Measurement/Management in Financial Markets. 4 Credits.

This course applies advanced mathematical techniques to the measurement, analysis, and management of risk. The focus is on financial risk. Sources of risk for financial instruments (e.g., market risk, interest rate risk, credit risk) are analyzed; models for these risk factors are studied and the limitation, shortcomings and compensatory techniques are addressed. Recommended Course Background: EN.553.644.

**Prerequisite(s):** Students may receive credit for EN.550.446/EN.553.446 or EN.553.646, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.647. Quantitative Portfolio Theory and Performance Analysis. 4 Credits.

This course focuses on modern quantitative portfolio theory, models, and analysis. Topics include intertemporal approaches to modeling and optimizing asset selection and asset allocation; benchmarks (indexes), performance assessment (including, Sharpe, Treynor and Jenson ratios) and performance attribution; immunization theorems; alpha-beta separation in management, performance measurement and attribution; Replicating Benchmark Index (RBI) strategies using cash securities / derivatives; Liability-Driven Investment (LDI); and the taxonomy and techniques of strategies for traditional management: Passive, Quasi-Passive (Indexing) Semi-Active (Immunization & Dedicated) Active (Scenario, Relative Value, Total Return and Optimization). In addition, risk management and hedging techniques are also addressed.

**Prerequisite(s):** Students may receive credit for (EN.550.447 OR EN.553.447) OR EN.553.647, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.649. Advanced Equity Derivatives. 4 Credits.

This course will cover the pricing, trading and risk management of equity derivatives, with emphasis on more exotic derivatives such as path-dependent and multi-asset derivatives. The course will emphasize practical issues: students will build their own pricing and risk management tools, and gain experience simulating the dynamic hedging of a complex derivatives portfolio. Students will practice structuring and selling equity derivative products. Pricing issues such a model selection, unobservable input parameters and calibration will be discussed, and students will learn techniques to manage the often highly nonlinear and discontinuous risks associated with these products. The course will have a significant computing component: both in the classroom and as homework projects, students will use Excel, write VBA macros and write and call C++ routines in the Microsoft Windows environment (which is the most common computing environment used by the financial industry). Recommended Course Background: EN.553.444.

**Prerequisite(s):** Students may receive credit for EN.550.449/EN.553.449 or EN.553.649, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.650. Computational Molecular Medicine. 4 Credits.

Computational systems biology has emerged as the dominant framework for analyzing high-dimensional "omics" data in order to uncover the relationships among molecules, networks and disease. In particular, many of the core methodologies are based on statistical modeling, including machine learning, stochastic processes and statistical inference. We will cover the key aspects of this methodology, including measuring associations, testing multiple hypotheses, and learning predictors, Markov chains and graphical models. In addition, by studying recent important articles in cancer systems biology, we will illustrate how this approach enhances our ability to annotate genomes, discover molecular disease networks, detect disease, predict clinical outcomes, and characterize disease progression. Whereas a good foundation in probability and statistics is necessary, no prior exposure to molecular biology is required (although helpful). Recommended Course Background: EN.553.620 AND EN.553.630.

**Prerequisite(s):** Students may receive credit for EN.550.450/EN.553.450 or EN.553.650, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.653. Mathematical Game Theory. 4 Credits.

Mathematical analysis of cooperative and noncooperative games. Theory and solution methods for matrix game (two players, zero-sum payoffs, finite strategy sets), games with a continuum of strategies, N-player games, games in rule-defined form. The roles of information and memory. Selected applications to economic, recreational, and military situations. Prereq: Multivariable Calculus, probability, linear algebra. Recommended Course Background: (AS.110.202 OR AS.110.211) AND EN.553.620 AND (EN.553.291 OR AS.110.201 OR AS.110.212)

**Prerequisite(s):** Students may receive credit for EN.550.453/EN.553.453 or EN.553.653, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.661. Optimization in Finance. 4 Credits.

A survey of many of the more important optimization methods and tools that are found to be useful in financial applications. Recommended Course Background: EN.553.642 OR EN.553.644.

**Prerequisite(s):** Students may receive credit for EN.550.461/EN.553.461 or EN.553.661, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.662. Optimization in Data Science. 3 Credits.

The course provides foundations and algorithms addressing optimization problems in modern data science. It covers smooth unconstrained descent methods (including deterministic and stochastic gradient), smooth constrained optimization and some non-smooth situations in the convex case. Each of these optimization problems will be related to specific data science training algorithms (such as logistic regression, neural networks, support vector machines or lasso). Prerequisites include multivariable calculus and linear algebra. Homework will include some programming components and students will be expected to have basic proficiency in computer languages such as Python (preferred) or Matlab.Recommended Course Background: Multivariable Calculus and Linear algebra.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.663. Network Models in Operations Research. 4 Credits.

In-depth mathematical study of network flow models in operations research, with emphasis on combinatorial approaches for solving them. Introduction to techniques for constructing efficient algorithms, and to some related data structures, used in solving shortest-path, maximumvolume, flow, and minimum-cost flow problems. Emphasis on linear models and flows, with brief discussion of non-linear models and network design. Recommended Course Background: EN.553.361 OR EN.553.761 OR EN.553.661.

**Prerequisite(s):** Students may receive credit for EN.550.463/EN.553.463 or EN.553.663, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.665. Introduction to Convexity. 4 Credits.

Convexity is a simple mathematical concept that has become central in a diverse range of applications in engineering, science and business applications. Our main focus from the applications perspective will be the use of convexity within optimization problems, where convexity plays a key role in identifying the "easy" problems from the "hard" ones. The course will have an equal emphasis on expositing the rich mathematical structure of the field itself (properties of convex sets, convex functions, Helly-Caratheorody-Radon type theorems, polarity/duality, subdifferential calculus, polyhedral theory), and demonstrating how these ideas can be leveraged to model and solve optimization problems (via a detailed study of linear programming and basics of nonlinear convex optimization). Recommend Course Background: Familiarity with basic real analysis, linear algebra.

**Prerequisite(s):** Students may receive credit for EN.550.465 /EN.553.465 or EN.553.665, but not both.

#### EN.553.669. Large-Scale Optimization For Data Science. 3 Credits.

Optimization formulations and algorithms have long played a central role in data analysis and machine learning. In the era of big data, the need to solve large-scale optimization problems is ubiquitous in essentially all quantitative areas of human endeavor, including industry and science. This course is a mathematically rigorous and comprehensive introduction to the field of large-scale optimization for data science and machine learning, and is based on the latest results and insights. We discuss the most important algorithms in the area, with analysis of their convergence and complexity properties, as well as their practical implementations. Applications of the methods covered in the course can be found virtually in all fields of data science including text analysis, page ranking, speech recognition, image classification, finance and decision sciences. Prerequisites: background in Linear Algebra (or Computational Linear Algebra), Multivariable Calculus, Probability, and a basic knowledge of programming - experience with at least one high-level computing language (e.g.: Python, Matlab, Julia, C, ....).

Distribution Area: Quantitative and Mathematical Sciences

#### EN.553.671. Combinatorial Analysis. 4 Credits.

An introduction to combinatorial analysis at the graduate level. Meets concurrently with 553.471. Counting techniques: generating functions, recurrence relations, Polya's theorem. Combinatorial designs: Latin squares, finite geometries, balanced incomplete block designs. Emphasis on problem solving. Recommended Course Background: EN.553.291 or AS.110.201

**Prerequisite(s):** Students may receive credit for EN.550.471/EN.553.471 or EN.553.671, but not both.

#### EN.553.672. Graph Theory. 4 Credits.

Study of systems of "vertices" with some pairs joined by "edges." Theory of adjacency, connectivity, traversability, feedback, and other concepts underlying properties important in engineering and the sciences. Topics include paths, cycles, and trees; routing problems associated with Euler and Hamilton; design of graphs realizing specified incidence conditions and other constraints. Attention directed toward problem solving, algorithms, and applications. One or more topics taken up in greater depth. Recommended Course Background: (EN.553.291 OR AS.110.201 OR AS.110.212)

**Prerequisite(s):** Students may receive credit for EN.550.472/EN.553.472 or EN.553.672, but not both.

### EN.553.680. Numerical Linear Algebra. 4 Credits.

A course on computational linear algebra and applications. Topics include floating-point arithmetic algorithms and convergence Gaussian elimination for linear systems matrix decompositions (LU, Cholesky, QR) iterative methods for systems (Jacobi, Gauss Seidel) approximation of eigenvalues (power method, QR-algorithm) and also singular values and singular-value decomposition (SVD). Theoretical topics such as vector spaces, inner products norms, linear operators, matrix norms, eigenvalues and canonical forms of matrices (Jordan, Schur) are reviewed as needed. Matlab is used to solve all numerical exercises.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.681. Numerical Analysis. 4 Credits.

Brief review of topics in elementary numerical analysis such as floatingpoint arithmetic, Gaussian elimination for linear equations, interpolation and approximation. Core topics to be covered: numerical linear algebra including eigenvalue and linear least-squares problems, iterative algorithms for nonlinear equations and least squares problems, and convergence theory of numerical methods. Other possible topics: sparse matrix computations, numerical solution of partial differential equations, finite element methods, and parallel algorithms.Recommended Course Background: Multivariable calculus, linear algebra, and differential equations.

Prerequisite(s): Students may take only one of EN.550.681, EN.553.481, EN.553.681 or EN.553.781.

## EN.553.683. Numerical Methods for Partial Differential Equations. 4 Credits.

We discuss numerical methods for solving partial differential equations, explaining how solution methods must be appropriate to the mathematical structure of the equation. Specific topics will be Hyperbolic PDE's (CFL stability condition, characteristics, convergence, nonlinear conservation laws, shock capturing), Parabolic PDE's (boundary conditions, explicit and implicit discretizations, consistency, stability, and convergence, operator splitting), Elliptic PDE's (iterative methods, variational formulations). We shall focus mainly on finite-difference schemes, but other methods such as finite element, finite volume, spectral methods, Chebyshev polynomials, etc. may be discussed as time permits. All numerical methods will be illustrated with Matlab scripts. Prior knowledge of Numerical Linear Algebra is strongly recommended. A good introductory course in the mathematics of Partial Differential Equations would also be helpful, but this material will be reviewed as necessary.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.688. Computing for Applied Mathematics. 3 Credits.

The aim of this course is to develop students' programming skills for solving solve problems commonly encountered in applied mathematics contexts. Specific problems that arise in applications of mathematics and data science (e.g. from finance, data analysis, or the physical sciences) are used to motivate concepts, techniques, and paradigms related to computation and programming. The Python language as well as a large collection of packages will be introduced. Recommended Course Background: EN.553.310 OR EN.553.311 OR (EN.553.420 AND EN.553.430). Students should be comfortable using computers but no prior programming background is required.

**Prerequisite(s):** Students may receive credit for EN.553.488 or EN.553.688, but not both.

#### EN.553.691. Dynamical Systems. 4 Credits.

Mathematical concepts and methods for describing and analyzing linear and nonlinear systems that evolve over time. Topics include boundedness, stability of fixed points and attractors, feedback, optimality, Liapounov functions, bifurcation, chaos, and catastrophes. Examples drawn from population growth, economic behavior, physical and engineering systems. The main mathematical tools are linear algebra and basic differential equations.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

### EN.553.692. Mathematical Biology. 3 Credits.

This course will examine the mathematical methods relevant to modeling biological phenomena, particularly dynamical systems and probability. Topics include ordinary differential equations and their simulation; stability and phase plane analysis; branching processes; Markov chains; and stochastically perturbed systems. Biological applications will be drawn from population growth, predator-prey dynamics, epidemiology, genetics, intracellular transport, and neuroscience. Recommended Course Background: EN.553.620 AND (AS.110.201 OR AS.110.212) AND (AS.110.302 OR AS.110.306 OR EN.553.291)

**Prerequisite(s):** Students may receive credit for EN.550.492/EN.553.492 or EN.553.692, but not both.

Distribution Area: Natural Sciences, Quantitative and Mathematical Sciences

#### EN.553.693. Mathematical Image Analysis. 4 Credits.

This course gives an overview of various mathematical methods related to several problems encountered in image processing and analysis, and presents numerical schemes to address them. It will focus on problems like image denoising and deblurring, contrast enhancement, segmentation and registration. The different mathematical concepts shall be introduced during the course; they include in particular functional spaces such as Sobolev and BV, Fourier and wavelet transforms, as well as some notions from convex optimization and numerical analysis. Most of such methods will be illustrated with algorithms and simulations on discrete images, using MATLAB. Prerequisites : linear algebra, multivariate calculus, basic programming in MATLAB. Recommended Course Background: A solid foundation Multivariable Calculus, Linear Algebra, and Probability. Real Analysis may help too, but it is not necessary.

**Prerequisite(s):** Students may receive credit for EN.550.493/EN.553.493 or EN.553.693, but not both.

Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.694. Applied and Computational Multilinear Algebra. 3 Credits.

In this seminar we plan to discuss generalizations of theorems and algorithms from matrix theory to hypermatrices. More specifically the seminar will discuss hypermatrix/tensor algebras, rank, spectra and transforms. Using the python friendly free open-source mathematics software SageMath and the hypermatrix algebra package we will discuss applications of hypermatrices to combinatorics, machine learning and data analysis. Preliminary knowledge of the Python language is not required. Recommended Course Background: AS.110.212 OR AS.110.201 OR EN.553.291.

**Prerequisite(s):** Students may receive credit for EN.553.494 or EN.553.694, but not both.

## EN.553.701. Real Analysis: Preparation for the Ph.D. Introductory Examination. 4 Credits.

This course is designed to prepare students for the Real Analysis part of the introductory exam of the Department of Applied Mathematics and Statistics. In this course we will cover fundamental topics in real analysis, such as, Set Theory, The Topology of Euclidean Space, Continuous Mappings, Uniform Convergence, Differentiable Mappings, Inverse & Implicit Function Theorems, Integration Theory, Fourier Series, and Basics of Differential Equations.

## EN.553.720. Probability Theory I. 4 Credits.

The course objectives are to develop probabilistic reasoning and problem solving approaches, to provide a rigorous mathematical basis for probability theory, and to examine several important results in the theory of probability. Topics include axiomatic probability, independence, random variables and their distributions, expectation, integration, variance and moments, probability inequalities, and modes of convergence of random variables. The course will include introductory measure theory as needed. Students are expected to have previous study of both analysis and probability. This course is the first half of a yearlong sequence. The second semester's course, EN.553.721 Probability Theory II, will cover classical limit theorems, characteristic functions, and conditional expectation. Prerequisite: real analysis (AS.110.405/ AS.110.415)

**Prerequisite(s):** Students may take EN.550.620 or EN.553.720, but not both.

### EN.553.721. Probability Theory II. 3 Credits.

Probability at the level of measure theory, focusing on limit theory. Modes of convergence, Poisson convergence, three-series theorem, strong law of large numbers, continuity theorem, central limit theory, Berry-Esseen theorem, infinitely divisible and stable laws. Recommended Course Background: EN.553.720 AND (AS.110.405 OR AS.110.415)

## EN.553.724. Probabilistic Machine Learning. 4 Credits.

Probabilistic machine learning harnesses the power of probability theory to provide models for complex data, as well as the algorithms that enable learning, inference, sampling, and decision-making for these models. The first part of the course will cover classical approaches based on directed and undirected probabilistic graphical models, including latent variable models and temporal models. We develop a toolkit of algorithms used for learning and inference, including message passing algorithms, Markov chain Monte Carlo, and variational inference. Building on these foundational ideas, the second part of the course will cover modern (deep learning) approaches to generative modeling such as variational auto-encoders, generative adversarial networks, normalizing flows, and diffusion models. A background in machine learning is recommended. **Prerequisite(s):** EN.553.420 OR EN.553.620 OR EN.553.720 Distribution Area: Engineering, Quantitative and Mathematical Sciences

#### EN.553.728. Optimal Transport. 3 Credits.

This course is designed to cover recent results in Optimal Transport from an applied mathematical perspective. We will briefly start by covering the mathematical formulations required to understand basic applications during the first few weeks, but the majority of the class will consist of reading and presenting papers in the student's fields of interest that have significant overlap with Optimal Transport, both theoretical and applied. Faculty from JHU (and possibly other institutions) will occasionally present on how OT features in their research as well. Among other topics, we will focus on applications to Manifold Learning. The structure of this class is modeled after Equivariant Machine Learning (EN.553.743.01.FA22).

### EN.553.729. Topics in Probability. 3 Credits.

This seminar course will discuss the "probabilistic method," with applications to random graphs and percolation theory. Topics include linearity of expectation, first and second moment methods, the local lemma, correlation inequalities, martingale concentration results, the evolution of random graphs, Poisson approximation, stochastic ordering, bond and site percolation models, and the substitution method for bounding percolation thresholds. Students will present at least two short talks on relevant topics or applications of their choice. Prerequisites: 553.620 Introduction to Probability and 553.672 Graph Theory, or equivalents. No auditors permitted.

### EN.553.730. Statistical Theory I. 4 Credits.

The fundamentals of mathematical statistics will be covered. Topics include: distribution theory for statistics of normal samples, exponential statistical models, the sufficiency principle, least squares estimation, maximum likelihood estimation, uniform minimum variance unbiased estimation, hypothesis testing, the Neyman-Pearson lemma, likelihood ratio procedures, the general linear model, the Gauss-Markov theorem, simultaneous inference, decision theory, Bayes and minimax procedures, chi-square methods, goodness-of-fit tests, and nonparametric and robust methods.

**Prerequisite(s):** Students may take EN.550.630 or EN.553.730, but not both.

### EN.553.731. Statistical Theory II. 3 Credits.

Advanced concepts and tools fundamental to research in mathematical statistics and statistical inference: asymptotic theory;optimality; various mathematical foundations.

**Prerequisite(s):** EN.553.730 OR EN.553.720 OR (AS.110.405 AND EN.553.430) Knowledge of real analysis is required.

## EN.553.733. Nonparametric Bayesian Statistics. 3 Credits.

This course covers advanced topics in Bayesian statistical analysis beyond the introductory course. Therefore knowledge of basic Bayesian statistics is assumed (at the level of "A first course in Bayesian statistical methods", by Peter Hoff (Springer, 2009). The models and computational methods will be introduced with emphasis on applications to real data problems. This course will cover nonparametric Bayesian models including Gaussian process, Dirichlet process (DP), Polya trees, dependent DP, Indian buffet process, etc. Recommended Course Background: EN.553.432 or EN.553.632 or EN.553.732 or permission from the instructor

## EN.553.735. Topics in Statistical Pattern Recognition. 3 Credits.

The Dissimilarity Representation for Pattern Recognition. This course will investigate aspects of statistical inference and statistical pattern recognition associated with observing only dissimilarites between entities rather than observing feature vectors associated with the individual entities themselves.

## EN.553.736. System Identification and Likelihood Methods. 2 Credits.

The focus of this roundtable-format course will be stochastic modeling as relates to system identification and maximum likelihood. The principles and algorithms being covered in this course have tremendous importance in the world at large. For example, maximum likelihood is arguably the most popular method for parameter estimation in most real-world applications. System identification is the term used in many fields to refer to the process of mathematical model building from experimental data, with a special focus on dynamical systems. The system identification process refers to several important aspects of model building, including selection of the model form (linear or nonlinear, static or dynamic, etc.), experimental design, parameter estimation, and model validation. This course will cover topics such as the maximum likelihood formulation and theory for dynamical systems, the EM (expectation-maximization) algorithm and its variants, Fisher information, common model structures, online versus offline estimation, the role of feedback in identification (i.e., open-loop versus closed-loop estimation), standard and extended Kalman filtering, and uncertainty characterization (e.g., confidence regions). Recommended Course Background: Undergraduate-level matrix theory and ordinary differential equations; graduate-level course in probability and statistics (e.g., 553.430 or equivalent; in particular, students should have prior exposure to maximum likelihood and Bayes' rule). Prior experience in data analysis and algorithms will be helpful.

## EN.553.738. High-Dimensional Approximation, Probability, and Statistical Learning. 3 Credits.

The course covers fundamental mathematical ideas for certain approximation and statistical learning problems in high dimensions. We start with basic approximation theory in low-dimensions, in particular linear and nonlinear approximation by Fourier and wavelets in classical smoothness spaces, and discuss applications in imaging, inverse problems and PDE's. We then introduce notions of complexity of function spaces, which will be important in statistical learning. We then move to basic problems in statistical learning, such as regression and density estimation. The interplay between randomness and approximation theory is introduced, as well as fundamental tools such as concentration inequalities, basic random matrix theory, and various estimators are constructed in detail, in particular multi scale estimators. At all times we consider the geometric aspects and interpretations, and will discuss concentration of measure phenomena, embedding of metric spaces, optimal transportation distances, and their applications to problems in machine learning such as manifold learning and dictionary learning for signal processing.

## EN.553.739. Statistical Pattern Recognition Theory & Methods. 3 Credits.

This biennial course covers topics in the theory, methods, and applications of machine learning from an explicitly statistical perspective. Recommended Course Background: (EN.550.420 OR EN.553.420 OR EN.553.620) AND (EN.550.430 OR EN.553.430 OR EN.553.630)

#### EN.553.740. Machine Learning I. 3 Credits.

This course is the first part of a two-semester sequence that focuses on theoretical and practical aspects of statistical learning. After introducing background material on inner-product spaces, reproducing kernels and on optimization, the course discusses fundamental concepts of machine learning (such as generalization error, Bayes estimators and the bias vs. variance dilemma) and studies a collection of learning algorithms for classification and regression. The topics that are discussed include linear and kernel regression, support vector machines, lasso, logistic regression, decision trees and neural networks. Students will need a solid background in multivariate calculus, linear algebra, probability and statistics to complete the course.Recommended Course background: 553.620 and 553.630 or higher, and prerequisites for these courses.

### EN.553.741. Machine Learning II. 3 Credits.

This course is the second part of a two-semester sequence that focuses on theoretical and practical aspects of statistical learning. The course will have two distinct parts. The first one will discuss some fundamentals of statistical learning theory, including some concentration inequality, generalization bounds and VC dimension. The second one will introduce problems and algorithms for unsupervised data analysis, including dimension reduction, manifold learning and clustering. Recommended course background: 553.740.

#### EN.553.742. Statistical Inference on Graphs. 3 Credits.

This course provides an introduction to and overview of current research in random graph inference, with a particular focus on spectral methods and their applications to inference for independent-edge random graphs. Topics include concentration inequalities; analysis of matrix perturbations; spectral decompositions of graph adjacency and Laplacian matrices; consistent estimation of latent variables associated to vertices; clustering, community detection, and classification in networks; and multi-sample hypothesis testing for graphs. Emphasis will be on a framework for establishing classical properties---consistency, normality, and efficiency---for estimators of graph parameters. Students will read papers in the literature and are expected to participate actively in class. Recommended prerequisites EN.553.792 and EN.553.630.

### EN.553.743. Equivariant Machine Learning. 3 Credits.

This is a graduate course in the topic of equivariant machine learning and graph neural networks. The course will have a fixed schedule with a preselected list of theoretical research papers to discuss each class (2.5 hours once a week). Each week two students will present one paper to the class and discussion will follow. The evaluation will be based on the quality and clarity of the presentations and in-class participation. There will be no homework nor exams.Prerequisites include basic knowledge of machine learning and probability.

### EN.553.749. Advanced Financial Theory. 4 Credits.

The first part of the course will review in depth the main instruments in the various asset classes, as well as the founding results on investment decision, capital budgeting and project financing. The second part will analyze the theory of the firm: capital structure, dilution and share repurchase, dividend policy, Modigliani- Miller theorem and will lead to the contingent claim pricing of corporate debt and equity as in Merton (1974) and its extensions . The third part will extend the CAPM to the Arbitrage Pricing Theory of Ross (1976) and its theoretical and operational consequences. The fourth part will be dedicated to the stochastic modelling of the yield curve to price caps, floors and swaptions, and their use in the Asset Liability Management of a bank and insurance company. This course will not begin until mid-October.

# EN.553.753. Three Key Aspects of Climate Change: Energy Transition, Critical Metals and Sustainable Agriculture. 3 Credits.

Energy markets and energy derivatives are unique and require special attention and special risk management tools. Topics include: Shipping Markets, Freight Indexes, Forward Freight Agreements, and Supply Chain. Coal markets across the world and their Rebirth Metal physical markets, major Exchanges (LME, SHFE), and the role of Metals in the development of Renewables, particularly Steel, Aluminum, Palladium. Agricultural (grains and softs) markets, Biofuels, fertilizers and water. Examples of forward curves and their different shapes; examples of volatility skews. Relationship between price volatility and Inventory ; Theory of Storage Usefulness of Asian options in Commodities and Valuation. Commodity Indexes and the Different Ways of Investing in Commodities.

### EN.553.761. Nonlinear Optimization I. 3 Credits.

This course considers algorithms for solving various nonlinear optimization problems and, in parallel, develops the supporting theory. The primary focus will be on unconstrained optimization problems. Topics for the course will include: necessary and sufficient optimality conditions; steepest descent method; Newton and quasi-Newton based line-search, trust-region, and adaptive cubic regularization methods; linear and nonlinear least-squares problems; linear and nonlinear conjugate gradient methods. Recommended Course Background: Multivariable Calculus, Linear Algebra, Real Analysis such as AS.110.405 **Prerequisite(s):** Students may take EN.550.661 or EN.553.761, but not both.

## EN.553.762. Nonlinear Optimization II. 3 Credits.

This course considers algorithms for solving various nonlinear optimization problems and, in parallel, develops the supporting theory. The primary focus will be on constrained optimization problems. Topics for the course will include: necessary and sufficient optimality conditions for constrained optimization; projected-gradient and two-phase accelerated subspace methods for bound-constrained optimization; simplex and interior-point methods for linear programming; duality theory; and penalty, augmented Lagrangian, sequential quadratic programming, and interior-point methods for general nonlinear programming. In addition, we will consider the Alternating Direction Method of Multipliers (ADMM), which is applicable to a huge range of problems including sparse inverse covariance estimation, consensus, and compressed sensing. Recommended Course Background: Multivariable Calculus, Linear Algebra, Real Analysis such as AS.110.405.

## EN.553.763. Stochastic Search and Optimization. 3 Credits.

An introduction to stochastic search and optimization, including discrete and continuous optimization problems. Topics will include the "no free lunch" theorems, beneficial effects of injected Monte Carlo randomness, algorithms for global and local optimization problems, random search, recursive least squares, stochastic approximation, simulated annealing, evolutionary and genetic algorithms, and statistical multiple comparisons. Recommended Course Background: Graduate course in probability and statistics and knowledge of basic matrix algebra.

## EN.553.764. Modeling, Simulation, and Monte Carlo. 3 Credits.

Concepts and statistical techniques critical to constructing and analyzing effective simulations; emphasis on generic principles rather than specific applications. Topics include model building (bias-variance tradeoff, model selection,, Fisher information), benefits and drawbacks of simulation modeling, random number generation, simulation-based optimization, discrete multiple comparisons using simulations, Markov chain Monte Carlo (MCMC), and input selection using optimal experimental design.

#### EN.553.766. Combinatorial Optimization. 3 Credits.

The main goal of this course is to introduce students to combinatorial optimization techniques. The first part of the course will focus on combinatorial algorithms for classical problems. The next part of the course will show how polyehdral theory can be used to deal with combinatorial optimization problems in a unifying manner. Familiarity with linear programming and algorithms desirable but not strictly required. Recommended Course Background: Linear Algebra.

## EN.553.767. Iterative Algorithms in Machine Learning: Theory and Applications. 3 Credits.

This course teaches an overview of modern (randomized) iterative methods for applications inmachine learning and data science. In particular, we will discuss the theoretical basics of stochastic optimization, iterative algorithms for variational inequalities, scalability of algorithms to large datasets, and challenges in distributed optimization, such as decentralized or federated machine learning. We will cover a set of foundational papers and a selection of recent publications. Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.780. Shape and Differential Geometry. 3 Credits.

The purpose of this class is to provide an elementary knowledge of the differential geometry of curves and surfaces, and to place this in relation with the description and characterization of 2D and 3D shapes. Intrinsic local and semi-local descriptors, like the curvature or the second fundamental form will be introduced, with an emphasis on the invariance of these features with respect to rotations, translations, etc. Extension of this point of view to other class of linear transformations will be given, as well as other types of shape descriptors, like moments or medial axes. Recommended Course Background: Calculus III and linear algebra Distribution Area: Engineering, Quantitative and Mathematical Sciences

## EN.553.782. Statistical Uncertainty Quantification. 3 Credits.

This course introduces uncertainty quantification (UQ) on mathematical models and data, with emphasis on the use of stochastic processes and probability theory. Topics include computer experiments, designs, conditional probability, Bayesian inference, Gaussian stochastic processes, continuity, reproducing kernel Hilbert space, covariance functions, computer model emulation, parameter estimation, approximation, dynamic linear models, Kalman filter, computation, sensitivity analysis, functional ANOVA, model selection and calibration. Examples of some continuous time processes will be introduced, such as Brownian motion, Brownian bridge, O-U process, with extensions to multidimensional input space. Uncertainty analysis of mathematical models will be the focus from both theoretical and computational perspectives. Applications will concentrate on understanding and predicting the behavior of complex systems in science and engineering. Prerequisite EN.553.620 or EN.553.720 Recommended course background: EN.553.630 or EN.553.730.

**Prerequisite(s):** Students may take EN.550.782 or EN.553.782, but not both.

## EN.553.784. Mathematical Foundations of Computational Anatomy. 3 Credits.

The course will provide fundamental concepts and methods that pertain the analysis of the variation of anatomical shapes extracted from medical images. It will review basic properties of the most important shape representations (landmark, curves, surfaces, images...), describe distances and discrepancy measures that allow for their comparison, and introduce nonlinear optimal control methods that underlie the Large Deformation Diffeomorphic Metric Mapping (LDDMM) family of registration algorithms. The course will then discuss shape averaging methods and template-centered representations for the analysis of shape datasets.Recommended Course Background: Optimization (EN.553.361 or higher) and (AS.110.202 OR AS.110.211 or higher) AND AS.110.302 or higher.

## EN.553.785. Asymptotic Analysis. 3 Credits.

Asymptotic analysis is a branch of mathematics that emphasizes finding approximate solutions for either small or large parameters, which have many benefits in various scientific and engineering disciplines. This is because, due to the complexity and mathematical formulation of the problem, analytical solutions are either difficult to obtain or impractical. The goal of this course is to introduce students to some of the most frequently used methods consisting of the following main components. First, an introduction to asymptotic sequences and expansions will be provided as well as some common techniques to obtaining asymptotic expansions on integrals. Next, some common transforms and their inverses will be introduced as well as techniques on finding the asymptotic representation of their inverses. Finally, we will examine some techniques for finding asymptotic representations of solutions resulting from ordinary differential equations. Throughout this course students will also be introduced to some special functions as practical examples to demonstrate how these techniques can be applied to provide robust approximations. Recommended Course Background: Differential Equations and either of the following courses AS.110.405, 110.311, or 110.607.

## EN.553.790. Neural Networks and Feedback Control Systems. 2 Credits.

This roundtable course is an introduction to two related areas?neural networks (NNs) and control systems based on the use of feedback. Artificial NNs are effective conceptual and computational vehicles for many important applications; feedback control is relevant to virtually all natural and human-made systems. NNs are applied in areas such as system modeling and control, function approximation, time-series filtering/prediction/smoothing, speech/image/signal processing, and pattern recognition. Topics to be covered for NNs include network architecture, learning algorithms, and applications. Specific NNs discussed include perceptrons, feedforward networks with backpropagation, and recurrent networks. This course also provides an introduction to feedback control systems, including the role of feedback in regulating systems and in achieving stability in systems. We consider stochastic (noise) effects in feedback systems. We also consider the interface of NNs and control by discussing how NNs are used in building modern control systems in problems where standard methods are infeasible. Recommended Course Background: Matrix theory, differential equations, and a graduate course in probability and statistics.

**EN.553.791.** Internship - Financial Mathematics. 2 Credits. This course is open only to AMS department master's students.

#### EN.553.792. Matrix Analysis and Linear Algebra. 4 Credits.

A second course in linear algebra with emphasis on topics useful in analysis, economics, statistics, control theory, and numerical analysis. Review of linear algebra, decomposition and factorization theorems, positive definite matrices, norms and convergence, eigenvalue location theorems, variational methods, positive and nonnegative matrices, generalized inverses. Prerequisite: one semester of real analysis. **Prerequisite(s):** Students may take EN.550.692 or EN.553.792, but not both.

#### EN.553.793. Turbulence Theory. 3 Credits.

An advanced introduction to turbulence theory for graduate students in the physical sciences, engineering and mathematics. Both intuitive understanding and exact analysis of the fluid equations will be stressed. Previous familiarity with fluid mechanics is not required, although it could be helpful.

#### EN.553.794. Turbulence Theory II. 3 Credits.

This course will continue the theoretical investigation of fluid turbulence, directly following on from EN.550.693. Topics to be considered are turbulent vortex dynamics, Lagrangian dynamics, and special topics such as wall-bounded turbulence, free shear flows, two-dimensional and quasigeostrophic turbulence, MHD turbulence, etc. Cross-listed with Physics

## EN.553.795. Matrix Analysis and Linear Algebra II. 3 Credits.

Additional topics in linear algebra, with emphasis on ideas useful in analysis, economics, statistics, control theory, and numerical analysis, and building on the material of EN.553.792. Singular value inequalities, perturbation of singular subspaces and eigenspaces, field of values, inertia, Kronecker and Hadamard products, matrix equations, matrix functions. Prerequisites: EN.553.792 **Prerequisite(s):** EN.553.792

Distribution Area: Engineering, Quantitative and Mathematical Sciences

# EN.553.797. Introduction to Control Theory and Optimal Control. 3 Credits.

A control system is a dynamical system on which one can act through a parameter that can be chosen freely at any point in time. In this class, we will be interested in two main problems. The first one is controllability, which studies conditions for the existence of controls allowing an initial point to be driven to any other point. The second one is optimal control, in which we will study methods to minimize a certain cost over all possible controls, possibly with endpoint constraints. Such problems have many applications in engineering: crossing a river with minimal fuel, planning trajectories of rocket engines etc. Recommended Course Background: Multivariate Calculus, Linear Algebra, Differential Equations. Some familiarity with Optimization is recommended, but not mandatory.

## EN.553.799. Topics In Applied Math. 3 Credits.

Machine learning systems have huge capabilities, and they are increasingly being deployed in many real-world applications. Therefore, it is critical to make sure that they are safe and trustworthy. This course focuses on understanding aspects regarding fairness, privacy, explainability, and robustness of machine learning models. The course will cover a list of recent research papers in the field, featuring practical aspects as well as mathematical aspects of these topics. The course not only focuses on the theory of fairness, privacy, explainability, and robustness of machine learning models, but also it aims to develop students' communication skills.

#### EN.553.800. Dissertation Research. 3 - 20 Credits.

Reading, research, or project work for advanced graduate students. Arranged individually between students and faculty.

## EN.553.801. Department Seminar. 1 Credit.

A variety of topics discussed by speakers from within and outside the university. Required of all resident department graduate students.

#### EN.553.802. Graduate Independent Study. 3 - 5 Credits.

Graduate independent study for Applied Mathematics & Statistics department students.

## EN.553.804. Approved External Coursework. 3 - 20 Credits.

The JHU Financial Mathematics Master's program requires that students satisfy a summer internship requirement. Students unable to secure an internship, may satisfy the requirements by developing a course plan which meets with the approval of the advisor. Course plans chosen must be designed to improve knowledge and skills that will ultimately make the student more attractive to potential employers, and build upon one or more of the following areas of focus: finance, natural language processing, machine learning, particular coding languages, and cloud computing.Students may choose from the following options:• Coursera (with certificate of completion)• Code Academy• LeetCode• LinkedIn LearningThe external course plan must be approved by student's advisor prior to enrollment. Students are required to document experiences and submit a report on the skills and knowledge they developed taking those approved external courses. Final reports will be graded Pass/Fail.

## EN.553.806. Capstone Experience in Data Science. 3 - 10 Credits.

Project work for Data Science Master's students. Arranged individually between students and faculty.

Distribution Area: Quantitative and Mathematical Sciences

## EN.553.809. Master's Research. 3 - 10 Credits.

Reading, research, or project work for Master's level students. Arranged individually between students and faculty.

## EN.553.847. Financial Mathematics Masters Seminar. 1 Credit.

This course is only open to students enrolled in the MSE in Financial Mathematics program. Advanced topics chosen according to the interests of the instructor and graduate students. The course will focus on recent research articles in the financial mathematics literature.

### EN.553.861. Nonsmooth Optimization Seminar. 3 Credits.

Readings and seminar in nonsmooth optimization. Topics may include nonsmooth, nonconvex analysis (generalized gradient and subdifferentials) and nonsmooth, nonconvex optimization methods.

## **Cross Listed Courses**

## **Computer Science**

## EN.601.442. Modern Cryptography. 3 Credits.

Modern Cryptography includes seemingly paradoxical notions such as communicating privately without a shared secret, proving things without leaking knowledge, and computing on encrypted data. In this challenging but rewarding course we will start from the basics of private and public key cryptography and go all the way up to advanced notions such as zeroknowledge proofs, functional encryption and program obfuscation. The class will focus on rigorous proofs and require mathematical maturity. **Prerequisite(s):** Students may receive credit for only one of EN.600.442, EN.601.442, EN.601.642.;(EN.601.230 OR EN.601.231) AND (EN.553.310 OR EN.553.311 OR EN.553.420 OR EN.553.421)

Distribution Area: Engineering, Quantitative and Mathematical Sciences AS Foundational Abilities: Science and Data (FA2)

#### EN.601.642. Modern Cryptography. 3 Credits.

Same material as 601.442, for graduate students. Modern Cryptography includes seemingly paradoxical notions such as communicating privately without a shared secret, proving things without leaking knowledge, and computing on encrypted data. In this challenging but rewarding course we will start from the basics of private and public key cryptography and go all the way up to advanced notions such as zero-knowledge proofs, functional encryption and program obfuscation. The class will focus on rigorous proofs and require mathematical maturity. Required course background: Probability & Automata/Computation Theory

**Prerequisite(s):** Students may receive credit for only one of EN.601.442 OR EN.601.642.

Distribution Area: Engineering, Quantitative and Mathematical Sciences