

AS.110 (MATHEMATICS)

AS.110.105. Precalculus. 4 Credits.

This course starts from scratch and provides students with all the background necessary for the study of calculus. It includes a review of algebra, trigonometry, exponential and logarithmic functions, coordinates and graphs. Each of these tools will be introduced in its cultural and historical context. The concept of the rate of change of a function will be introduced.

Area: Quantitative and Mathematical Sciences

AS.110.106. Calculus I (Biology and Social Sciences). 4 Credits.

Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Many applications to the biological and social sciences will be discussed.

Area: Quantitative and Mathematical Sciences

AS.110.107. Calculus II (For Biological and Social Science). 4 Credits.

Differential and integral Calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. Applications to the biological and social sciences will be discussed, and the courses are designed to meet the needs of students in these disciplines.

Area: Quantitative and Mathematical Sciences

AS.110.108. Calculus I (Physical Sciences & Engineering). 4 Credits.

Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor's theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.

Area: Quantitative and Mathematical Sciences

AS.110.109. Calculus II (For Physical Sciences and Engineering). 4 Credits.

Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor's theorem and applications, infinite sequences and series. Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.

Area: Quantitative and Mathematical Sciences

AS.110.113. Honors Single Variable Calculus. 4 Credits.

This is an honors alternative to the Calculus sequences AS.110.106-AS.110.107 or AS.110.108-AS.110.109 and meets the general requirement for both Calculus I and Calculus II (although the credit hours count for only one course). It is a more theoretical treatment of one variable differential and integral calculus and is based on our modern understanding of the real number system as explained by Cantor, Dedekind, and Weierstrass. Students who want to know the 'why's and how's' of Calculus will find this course rewarding. Previous background in Calculus is not assumed. Students will learn differential Calculus (derivatives, differentiation, chain rule, optimization, related rates, etc), the theory of integration, the fundamental theorem(s) of Calculus, applications of integration, and Taylor series. Students should have a strong ability to learn mathematics quickly and on a higher level than that of the regular Calculus sequences.

Area: Quantitative and Mathematical Sciences

AS.110.201. Linear Algebra. 4 Credits.

Vector spaces, matrices, and linear transformations. Solutions of systems of linear equations. Eigenvalues, eigenvectors, and diagonalization of matrices. Applications to differential equations.

Prerequisite(s): Grade of C- or better in AS.110.107 OR AS.110.109 OR AS.110.113 OR AS.110.202 OR AS.110.302, or a 5 on the AP BC exam.

Area: Quantitative and Mathematical Sciences

AS.110.202. Calculus III. 4 Credits.

Calculus of functions of more than one variable: partial derivatives, and applications; multiple integrals, line and surface integrals; Green's Theorem, Stokes' Theorem, and Gauss' Divergence Theorem.

Prerequisite(s): Grade of C- or better in AS.110.107 OR AS.110.109 OR AS.110.113 OR AS.110.201 OR AS.110.212 OR AS.110.302, or a 5 or better on the AP BC exam.

Area: Quantitative and Mathematical Sciences

AS.110.211. Honors Multivariable Calculus. 4 Credits.

This course includes the material in AS.110.202 with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or especially mathematics.

AS.110.211-AS.110.212 used to be an integrated yearlong course, but now the two are independent courses and can be taken in either order.

Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)

Area: Quantitative and Mathematical Sciences

AS.110.212. Honors Linear Algebra. 4 Credits.

This course includes the material in AS.110.201 with additional applications and theory, and is recommended only for mathematically able students majoring in physical science, engineering, or mathematics who are interested in a proof-based version of linear algebra. This course can serve as an Introduction to Proofs (IP) course. Prerequisites: Grade of B+ or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam.

Area: Quantitative and Mathematical Sciences.

Prerequisite(s): Grade of B+ or better in AS.110.107 or AS.110.109 or AS.110.113 or AS.110.202, or AS.110.302, or a 5 on the AP BC exam.

Area: Quantitative and Mathematical Sciences

AS.110.225. Problem Solving Lab. 2 Credits.

This course is an introduction to mathematical reason and formalism in the context of mathematical problem solving, such as induction, invariants, inequalities and generating functions. This course does not satisfy any major requirement, and may be taken more than once for credit. It is primarily used as training for the William Lowell Putnam Mathematics Competition.

Area: Quantitative and Mathematical Sciences

AS.110.301. Introduction to Proofs. 4 Credits.

This course will provide a practical introduction to mathematical proofs with the aim of developing fluency in the language of mathematics, which itself is often described as "the language of the universe." Along with a library of proof techniques, we shall tour propositional logic, set theory, cardinal arithmetic, and metric topology and explore "proof relevant" mathematics by interacting with a computer proof assistant. This course on the construction of mathematical proof will conclude with a deconstruction of mathematical proof, interrogating the extent to which proof serves as a means to discover universal truths and assessing the mechanisms by which the mathematical community achieves consensus regarding whether a claimed result has been proven.

Area: Quantitative and Mathematical Sciences

AS.110.302. Differential Equations and Applications. 4 Credits.

This is a course in ordinary differential equations (ODEs), equations involving an unknown function of one independent variable and some of its derivatives, and is primarily a course in the study of the structure of and techniques for solving ODEs as mathematical models. Specific topics include first and second ODEs of various types, systems of linear differential equations, autonomous systems, and the qualitative and quantitative analysis of nonlinear systems of first-order ODEs. Laplace transforms, series solutions and the basics of numerical solutions are included as extra topics. Prerequisites: Grade of C- or better in 110.107 or 110.109 or 110.113, or a 5 on the AP BC exam. Area: Quantitative and Mathematical Sciences.

Prerequisite(s): Grade of C- or better in AS.110.107 or AS.110.109 or AS.110.113 or AS.110.201 or AS.110.202 or AS.110.211 or AS.110.212, or a 5 on the AP BC exam.

Area: Quantitative and Mathematical Sciences

AS.110.304. Elementary Number Theory. 4 Credits.

The student is provided with many historical examples of topics, each of which serves as an illustration of and provides a background for many years of current research in number theory. Primes and prime factorization, congruences, Euler's function, quadratic reciprocity, primitive roots, solutions to polynomial congruences (Chevalley's theorem), Diophantine equations including the Pythagorean and Pell equations, Gaussian integers, Dirichlet's theorem on primes.

Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)

Area: Quantitative and Mathematical Sciences

AS.110.311. Methods of Complex Analysis. 4 Credits.

This course is an introduction to the theory of functions of one complex variable. Its emphasis is on techniques and applications, and it serves as a basis for more advanced courses. Functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions.

Prerequisite(s): Grade of C- or better in (AS.110.202 or AS.110.211)

Area: Quantitative and Mathematical Sciences

AS.110.328. Non-Euclidean Geometry. 4 Credits.

For 2,000 years, Euclidean geometry was the geometry. In the 19th century, new, equally consistent but very different geometries were discovered. This course will delve into these geometries on an elementary but mathematically rigorous level.

Area: Quantitative and Mathematical Sciences

AS.110.375. Introduction to Mathematical Cryptography. 4 Credits.

An Introduction to Mathematical Cryptography is an introduction to modern cryptography with an emphasis on the mathematics behind the theory of public key cryptosystems and digital signature schemes. The course develops the mathematical tools needed for the construction and security analysis of diverse cryptosystems. Other topics central to mathematical cryptography covered are: classical cryptographic constructions, such as Diffie-Hellman key exchange, discrete logarithm-based cryptosystems, the RSA cryptosystem, and digital signatures. Fundamental mathematical tools for cryptography studied include: primality testing, factorization algorithms, probability theory, information theory, and collision algorithms. A survey of important recent cryptographic innovations, such as elliptic curves, elliptic curve and pairing-based cryptography are included as well. This course is an ideal introduction for mathematics and computer science students to the mathematical foundations of modern cryptography.

Area: Quantitative and Mathematical Sciences

AS.110.401. Introduction to Abstract Algebra. 4 Credits.

An introduction to the basic notions of modern abstract algebra and can serve as an Introduction to Proofs (IP) course. This course is an introduction to group theory, with an emphasis on concrete examples, and especially on geometric symmetry groups. The course will introduce basic notions (groups, subgroups, homomorphisms, quotients) and prove foundational results (Lagrange's theorem, Cauchy's theorem, orbit-counting techniques, the classification of finite abelian groups). Examples to be discussed include permutation groups, dihedral groups, matrix groups, and finite rotation groups, culminating in the classification of the wallpaper groups. Prerequisites: Grade of C- or better in 110.201 or 110.212 Area: Quantitative and Mathematical Sciences.

Prerequisite(s): Grade of C- or better in (AS.110.201 or AS.110.212)

Area: Quantitative and Mathematical Sciences

AS.110.405. Real Analysis I. 4 Credits.

This course is designed to give a firm grounding in the basic tools of analysis. It is recommended as preparation (but may not be a prerequisite) for other advanced analysis courses and may be taken as an Introduction to Proofs (IP) course. Topics include the formal properties of real and complex number systems, topology of metric spaces, limits, continuity, infinite sequences and series, differentiation, Riemann-Stieltjes integration. Prerequisites: Grade of C- or better in 110.201 or 110.212 and 110.202 or 110.211

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)

Area: Quantitative and Mathematical Sciences

AS.110.406. Real Analysis II. 4 Credits.

This course continues AS.110.405 with an emphasis on the fundamental notions of modern analysis. Sequences and series of functions, Fourier series, equicontinuity and the Arzela-Ascoli theorem, the Stone-Weierstrass theorem, functions of several variables, the inverse and implicit function theorems, introduction to the Lebesgue integral.

Area: Quantitative and Mathematical Sciences

AS.110.407. Honors Complex Analysis. 4 Credits.

AS.110.407. Honors Complex Analysis. 4.00 Credits. This course is an introduction to the theory of functions of one complex variable for honors students. Its emphasis is on techniques and applications, and can serve as an Introduction to Proofs (IP) course. Topics will include functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions, as well as applications to number theory and harmonic analysis. Area: Quantitative and Mathematical Sciences. This is not an Introduction to Proofs course (IP) and may not be taken as a first proof-based mathematics course except at the discretion of the instructor. This course satisfies a core requirement of the mathematics major as a second analysis course, and is a core requirement for honors in the major.

Prerequisite(s): AS.110.405 OR AS.110.415

Area: Quantitative and Mathematical Sciences

AS.110.411. Honors Algebra I. 4 Credits.

An introduction to the basic notions of modern algebra for students with some prior acquaintance with abstract mathematics. Elements of group theory: groups, subgroups, normal subgroups, quotients, homomorphisms. Generators and relations, free groups, products, abelian groups, finite groups. Groups acting on sets, the Sylow theorems. Definition and examples of rings and ideals.

Prerequisite(s): Grade of C- or better in AS.110.212 OR AS.110.304 OR AS.110.113 OR AS.110.405 OR AS.110.415 OR AS.110.407 OR AS.110.413 OR AS.110.421

Area: Quantitative and Mathematical Sciences

AS.110.412. Honors Algebra II. 4 Credits.

This is a continuation of 110.411 Honors Algebra I. Topics studies include principal ideal domains, structure of finitely generated modules over them. Introduction to field theory. Linear algebra over a field. Field extensions, constructible polygons, non-trisectability. Splitting field of a polynomial, algebraic closure of a field. Galois theory: correspondence between subgroups and subfields. Solvability of polynomial equations by radicals. Prerequisites: Grade of C- or better in 110.201 or 110.212. Area: Quantitative and Mathematical Sciences.

Prerequisite(s): C- or better in AS.110.411

Area: Quantitative and Mathematical Sciences

AS.110.413. Introduction To Topology. 4 Credits.

Topological spaces, connectedness, compactness, quotient spaces, metric spaces, function spaces. An introduction to algebraic topology: covering spaces, the fundamental group, and other topics as time permits.

Prerequisite(s): Grade of C- or better in (AS.110.202 OR AS.110.211)

Area: Quantitative and Mathematical Sciences

AS.110.415. Honors Analysis I. 4 Credits.

This highly theoretical sequence in analysis is reserved for the most able students. The sequence covers the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics.

Area: Quantitative and Mathematical Sciences

AS.110.416. Honors Analysis II. 4 Credits.

Lebesgue integration and differentiation. Elementary Hilbert and Banach space theory. Baire category theorem. Continuation of AS.110.415, introduction to real analysis.

Prerequisite(s): Grade of C- or better in AS.110.415

Area: Quantitative and Mathematical Sciences

AS.110.417. Partial Differential Equations. 4 Credits.

Characteristics. classification of second order equations, well-posed problems. separation of variables and expansions of solutions. The wave equation: Cauchy problem, Poisson's solution, energy inequalities, domains of influence and dependence. Laplace's equation: Poisson's formula, maximum principles, Green's functions, potential theory Dirichlet and Neumann problems, eigenvalue problems. The heat equation: fundamental solutions, maximum principles. Recommended Course Background: AS.110.405 or AS.110.415

Area: Quantitative and Mathematical Sciences

AS.110.421. Dynamical Systems. 4 Credits.

This is a course in the modern theory of Dynamical Systems. Topics include both discrete (iterated maps) and continuous (differential equations) dynamical systems and focuses on the qualitative structure of the system in developing properties of solutions. Topics include contractions, interval and planar maps, linear and nonlinear ODE systems including bifurcation theory, recurrence, transitivity and mixing, phase volume preservation as well as chaos theory, fractional dimension and topological entropy. May be taken as an Introduction to Proofs (IP) course. Prerequisites: Grade of C- or better in 110.201 or 110.212 OR 110.202 or 110.211 and 110.302 Area: Quantitative and Mathematical Sciences

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 or AS.110.211) AND 110.302

Area: Quantitative and Mathematical Sciences

AS.110.422. Representation Theory. 4 Credits.

This course will focus on the basic theory of representations of finite groups in characteristic zero: Schur's Lemma, Maschke's Theorem and complete reducibility, character tables and orthogonality, direct sums and tensor products. The main examples we will try to understand are the representation theory of the symmetric group and the general linear group over a finite field. If time permits, the theory of Brauer characters and modular representations will be introduced.

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.401 OR AS.110.411)

Area: Quantitative and Mathematical Sciences

AS.110.433. Introduction to Harmonic Analysis and Its Applications. 4 Credits.

The course is an introduction to methods in harmonic analysis, in particular Fourier series, Fourier integrals, and wavelets. These methods will be introduced rigorously, together with their motivations and applications to the analysis of basic partial differential equations and integral kernels, signal processing, inverse problems, and statistical/machine learning.

Prerequisite(s): (AS.110.201 OR AS.110.212 OR EN.550.291 OR EN.553.291) AND (AS.110.202 OR AS.110.211) AND (AS.110.405 OR AS.110.415)

Area: Quantitative and Mathematical Sciences

AS.110.435. Introduction to Algebraic Geometry. 4 Credits.

Algebraic geometry studies zeros of polynomials in several variables and is based on the use of abstract algebraic techniques, mainly from commutative algebra, for solving geometric problems about these sets of zeros. The fundamental objects of study are algebraic varieties which are the geometric manifestations of solutions of systems of polynomial equations. Algebraic geometry occupies a central place in modern mathematics and has multiple conceptual connections with diverse fields such as complex analysis, topology and number theory. This course aims to provide to an undergraduate student majoring in mathematics the fundamental background to approach the study of algebraic geometry by providing the needed abstract knowledge also complemented by several examples and applications.

Area: Quantitative and Mathematical Sciences

AS.110.439. Introduction To Differential Geometry. 4 Credits.

Theory of curves and surfaces in Euclidean space: Frenet equations, fundamental forms, curvatures of a surface, theorems of Gauss and Mainardi-Codazzi, curves on a surface; introduction to tensor analysis and Riemannian geometry; theorems egregium; elementary global theorems.

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)

Area: Quantitative and Mathematical Sciences

AS.110.441. Calculus on Manifolds. 4 Credits.

This course provides the tools for classical three-dimensional physics and mechanics. This course extends these techniques to the general locally Euclidean spaces (manifolds) needed for an understanding of such things as Maxwell's equations or optimization in higher dimensional contexts, eg. in economics. The course will cover the theory of differential forms and integration. Specific topics include Maxwell's equations in terms of 4D Lorentz geometry, vector (in particular, tangent) bundles, an introduction to de Rham theory, and Sard's theorem on the density of regular values of smooth functions. The course is intended to be useful to mathematics students interested in analysis, differential geometry, and topology, as well as to students in physics and economics.

Area: Quantitative and Mathematical Sciences

AS.110.443. Fourier Analysis. 4 Credits.

An introduction to the Fourier transform and the construction of fundamental solutions of linear partial differential equations. Homogeneous distributions on the real line: the Dirac delta function, the Heaviside step function. Operations with distributions: convolution, differentiation, Fourier transform. Construction of fundamental solutions of the wave, heat, Laplace and Schrödinger equations. Singularities of fundamental solutions and their physical interpretations (e.g., wave fronts). Fourier analysis of singularities, oscillatory integrals, method of stationary phase.

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212) AND (AS.110.202 OR AS.110.211)

Area: Quantitative and Mathematical Sciences

AS.110.445. Mathematical and Computational Foundations of Data Science. 4 Credits.

We will cover several topics in the mathematical and computational foundations of Data Science. The emphasis is on fundamental mathematical ideas (basic functional analysis, reproducing kernel Hilbert spaces, concentration inequalities, uniform central limit theorems), basic statistical modeling techniques (e.g. linear regression, parametric and non-parametric methods), basic machine learning techniques for unsupervised (e.g. clustering, manifold learning), supervised (classification, regression), and semi-supervised learning, and corresponding computational aspects (linear algebra, basic linear and nonlinear optimization to attack the problems above). Applications will include statistical signal processing, imaging, inverse problems, graph processing, and problems at the intersection of statistics/machine learning and physical/dynamical systems (e.g. model reduction for stochastic dynamical systems).

Area: Quantitative and Mathematical Sciences

AS.110.446. Introduction to Statistical Learning, Data Analysis and Signal Processing. 4 Credits.

Introduction to high dimensional data sets: key problems in statistical and machine learning. Geometric aspects. Principal component analysis, linear dimension reduction, random projections. Concentration phenomena: examples and basic inequalities. Metric spaces and embeddings thereof. Kernel methods. Nonlinear dimension reduction, manifold models. Regression. Vector spaces of functions, linear operators, projections. Orthonormal bases; Fourier and wavelet bases, and their use in signal processing and time series analysis. Basic approximation theory. Linear models, least squares. Bias and variance tradeoffs, regularization. Sparsity and compressed sensing. Multiscale methods. Graphs and networks. Random walks on graphs, diffusions, page rank. Block models. Spectral clustering, classification, semi-supervised learning. Algorithmic and computational aspects of the above will be consistently in focus, as will be computational experiments on synthetic and real data. Linear algebra will be used throughout the course, as will multivariable calculus and basic probability (discrete random variables). Basic experience in programming in C or MATLAB or R or Octave. ⁷Recommended Course Background: More than basic programming experience in Matlab or R; some more advanced probability (e.g. continuous random variables), some signal processing (e.g. Fourier transform, discrete and continuous). Co-listed with EN.553.416

Prerequisite(s): Grade of C- or better in (AS.110.201 OR AS.110.212)

Area: Quantitative and Mathematical Sciences

AS.110.503. Undergraduate Research in Mathematics. 1 - 4 Credits.

Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration > Online Forms.

AS.110.586. Independent Study. 0 - 4 Credits.

Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration > Online Forms.

AS.110.587. DRP Independent Study. 1 Credit.

Directed Reading Program (DRP) Independent Study.

Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration > Online Forms.

Area: Quantitative and Mathematical Sciences

AS.110.595. Internship. 1 Credit.

Prerequisite(s): You must request Independent Academic Work using the Independent Academic Work form found in Student Self-Service: Registration > Online Forms.

AS.110.599. Independent Study. 0 - 3 Credits.

Area: Quantitative and Mathematical Sciences

AS.110.601. Algebra.

An introductory graduate course on fundamental topics in algebra to provide the student with the foundations for number theory, algebraic geometry, and other advanced courses. Topics include group theory, commutative algebra, Noetherian rings, local rings, modules, rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras.

Area: Quantitative and Mathematical Sciences

AS.110.602. Algebra.

An introductory graduate course on fundamental topics in algebra to provide the student with the foundations for Number Theory, Algebraic Geometry, and other advanced courses. Topics include group theory, commutative algebra, Noetherian rings, local rings, modules, and rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras. Recommended Course Background: AS.110.401-AS.110.402

Area: Quantitative and Mathematical Sciences

AS.110.605. Real Variables.

Measure and integration on abstract and locally compact spaces (extension of measures, decompositions of measures, product measures, the Lebesgue integral, differentiation, L_p-spaces); introduction to functional analysis; integration on groups; Fourier transforms.

Area: Quantitative and Mathematical Sciences

AS.110.607. Complex Variables.

Analytic functions of one complex variable. Topics include Mittag-Leffler Theorem, Weierstrass factorization theorem, elliptic functions, Riemann-Roch theorem, Picard theorem, and Nevanlinna theory. Recommended Course Background: AS.110.311, AS.110.405

AS.110.608. Riemann Surfaces.

Abstract Riemann surfaces. Examples: algebraic curves, elliptic curves and functions on them. Holomorphic and meromorphic functions and differential forms, divisors and the Mittag-Leffler problem. The analytic genus. Bezout's theorem and applications. Introduction to sheaf theory, with applications to constructing linear series of meromorphic functions. Serre duality, the existence of meromorphic functions on Riemann surfaces, the equality of the topological and analytic genera, the equivalence of algebraic curves and compact Riemann surfaces, the Riemann-Roch theorem. Period matrices and the Abel-Jacobi mapping, Jacobi inversion, the Torelli theorem. Uniformization (time permitting).

AS.110.615. Algebraic Topology.

Polyhedra, simplicial and singular homology theory, Lefschetz fixed-point theorem, cohomology and products, homological algebra, Künneth and universal coefficient theorems, Poincaré and Alexander duality theorems.

AS.110.616. Algebraic Topology.

Polyhedra, simplicial and singular homology theory, Lefschetz fixed-point theorem, cohomology and products, homological algebra, Künneth and universal coefficient theorems, Poincaré; and Alexander duality theorems.

Area: Quantitative and Mathematical Sciences

AS.110.617. Number Theory.

Topics in advanced algebra and number theory, including local fields and adèles, Iwasawa-Tate theory of zeta functions and connections with Hecke's treatment, semisimple algebras over local and number fields, adèles geometry.

Area: Quantitative and Mathematical Sciences

AS.110.618. Number Theory.

Topics in advanced algebra and number theory, including local fields and adèles, Iwasawa-Tate theory of zeta-functions and connections with Hecke's treatment, semi-simple algebras over local and number fields, adèle geometry.

AS.110.619. Lie Groups and Lie Algebras.

Lie groups and Lie algebras, classification of complex semi-simple Lie algebras, compact forms, representations and Weyl formulas, symmetric Riemannian spaces.

Area: Quantitative and Mathematical Sciences

AS.110.631. Partial Differential Equations I.

An introductory graduate course in partial differential equations. Classical topics include first order equations and characteristics, the Cauchy-Kowalewski theorem, Laplace's equations, heat equation, wave equation, fundamental solutions, weak solutions, Sobolev spaces, maximum principles.

Prerequisite(s): Grade of C- or better in AS.110.605

AS.110.632. Partial Differential Equations II.

An introductory graduate course in partial differential equations. Classical topics include first order equations and characteristics, the Cauchy-Kowalewski theorem, Laplace's equation, heat equation, wave equation, fundamental solutions, weak solutions, Sobolev spaces, maximum principles. The second term focuses on special topics such as second order elliptic theory.

AS.110.633. Harmonic Analysis.

Fourier multipliers, oscillatory integrals, restriction theorems, Fourier integral operators, pseudodifferential operators, eigenfunctions. Undergrads need instructor's permission.

Area: Quantitative and Mathematical Sciences

AS.110.635. Microlocal Analysis.

Microlocal analysis is the geometric study of singularities of solutions of partial differential equations. The course will begin by introducing the geometric theory of (Schwartz) distributions: Fourier transform and Sobolev spaces, pseudo-differential operators, wave front set of a distribution, elliptic operators, Lagrangean distributions, oscillatory integrals, method of stationary phase, Fourier integral operators. The second semester will develop the theory and apply it to special topics such as asymptotics of eigenvalues/eigenfunctions of the Laplace operator on a Riemann manifold, linear and non-linear wave equation asymptotics of quantum systems, Bochner-Riesz means, maximal theorems.

AS.110.637. Functional Analysis.**AS.110.643. Algebraic Geometry.**

Affine varieties and commutative algebra. Hilbert's theorems about polynomials in several variables with their connections to geometry. General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, connections with complex analytic geometry and topology, schemes.

AS.110.644. Algebraic Geometry.

Affine varieties and commutative algebra. Hilbert's theorems about polynomials in several variables with their connections to geometry. General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch Theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, schemes, connections with complex analytic geometry and topology.

Area: Quantitative and Mathematical Sciences

AS.110.645. Riemannian Geometry.

Differential manifolds, vector fields, flows, Frobenius' theorem. Differential forms, deRham's theorem, vector bundles, connections, curvature, Chern classes, Cartan structure equations. Riemannian manifolds, Bianchi identities, geodesics, exponential maps. Geometry of submanifolds, hypersurfaces in Euclidean space. Other topics as time permits, e.g., harmonic forms and Hodge theorem, Jacobi equation, variation of arc length and area, Chern-Gauss-Bonnet theorems.

AS.110.646. Riemannian Geometry.

The goal is to give a self-contained course on mean curvature flow, starting with the basic linear heat equation in Euclidean space and – hopefully – getting to topics of current research. Mean curvature flow is a geometric heat equation that shares many properties with Ricci flow, harmonic map heat flow, Yang-Mills flow and the Navier-Stokes equations. Recommended Course Background: AS.110.605 and an undergraduate course in differential geometry; AS.110.645 and AS.110.631

AS.110.675. High-Dimensional Approximation, Probability, and Statistical Learning.

The course covers fundamental mathematical ideas for certain approximation and statistical learning problems in high dimensions. We start with basic approximation theory in low-dimensions, in particular linear and nonlinear approximation by Fourier and wavelets in classical smoothness spaces, and discuss applications in imaging, inverse problems and PDE's. We then introduce notions of complexity of function spaces, which will be important in statistical learning. We then move to basic problems in statistical learning, such as regression and density estimation. The interplay between randomness and approximation theory is introduced, as well as fundamental tools such as concentration inequalities, basic random matrix theory, and various estimators are constructed in detail, in particular multi scale estimators. At all times we consider the geometric aspects and interpretations, and will discuss concentration of measure phenomena, embedding of metric spaces, optimal transportation distances, and their applications to problems in machine learning such as manifold learning and dictionary learning for signal processing.

Area: Quantitative and Mathematical Sciences

AS.110.707. Functional Analysis.**AS.110.711. Topics in Topos Theory.**

Reading course to discuss Topics in Topos Theory

AS.110.712. Topics in Mathematical Physics.

AS.110.722. Topics in Homotopy Theory.

The course will focus on recent developments in homotopy theory, such as Galois theory for E_n ($n \geq 2$) ring-spectra, and on connections with number theory; in particular, work of Bhatt, Hesselholt, Lurie, Scholze and others on topological Hochschild homology and its applications to geometry over the p -adic complex numbers.

Area: Quantitative and Mathematical Sciences

AS.110.724. Topics in Arithmetic Geometry.

Topics around the subject of Arithmetic Geometry will be covered in this course.

Area: Quantitative and Mathematical Sciences

AS.110.726. Topics in Analysis.

The topics covered will involve the theory of calculus of Functors applied to Geometric problems like Embedding theory. Other related areas will be covered depending on the interest of the audience.

Area: Quantitative and Mathematical Sciences

AS.110.727. Topics in Algebraic Topology.**AS.110.728. Topics in Algebraic Topology.****AS.110.731. Topics in Geometric Analysis.****AS.110.733. Topics In Alg Num Theory.****AS.110.735. Topics In Hodge Theory.****AS.110.737. Topics Algebraic Geometry.****AS.110.738. Topics Algebraic Geometry.**

Introduction to toric varieties. This class is a general introduction to toric varieties. Toric varieties are special kinds of algebraic varieties which can be described by lattices and convex sets. They provide a rich source of concrete examples in complex geometry or mathematical physics. If time permits, we discuss in the end the stability of toric embeddings. Students should know basic notions of algebraic geometry (schemes, sheaves, linear systems), as covered in AS.110.643.

Area: Quantitative and Mathematical Sciences

AS.110.741. Topics in Partial Differential Equations.**AS.110.742. Topics In Partial Differential Equations.**

In this course we will be discussing some dispersive evolution equations, primarily the nonlinear Schrodinger equation. Topics will include well-posedness theory, conservation laws, and scattering. The course will be accessible to students who have not taken graduate partial differential equations or functional analysis.

AS.110.745. Introduction to Curvature Flows.**AS.110.749. Topics in Differential Geometry.**

In this class, we will study Aaron Naber and Jeff Cheeger's recent result on proving codimension four conjecture. We plan to talk about some early results of the structure on manifolds with lower Ricci bound by Cheeger and Colding. We will prove quantitative splitting theorem, volume convergence theorem, and the result that almost volume cone implies almost metric cone. Then we will discuss regularity of Einstein manifolds and the codimension four conjecture.

Area: Quantitative and Mathematical Sciences

AS.110.750. Topics in Representation Theory.

Area: Quantitative and Mathematical Sciences

AS.110.756. Topics in Algebra.

This will be a course in commutative algebra. Topics may include: Noetherian rings and modules, the Nullstellensatz, Hilbert basis theorem, localization, integrality, Noether normalization, primary decomposition, DVRs, Dedekind domains, dimension theory, smoothness and regularity, and homological methods.

Area: Quantitative and Mathematical Sciences

AS.110.757. Topics in Stochastic Dynamical Systems.

The course will present an introduction to stochastic dynamical systems and some applications in model reduction and data assimilation. The main focus will be on stability and ergodicity of stochastic dynamical systems, including stochastic differential equations driven by white and fractional noise, and their numerical approximations. We will then discuss model reduction, focusing on Mori-Zwanzig formalism and approximation of the generalized Langevin equation, and methods on the parametric inference of related stochastic systems. Data assimilation and stochastic control will also be briefly introduced.

Area: Quantitative and Mathematical Sciences

AS.110.790. Seminar in Complex Geometry.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.

Area: Quantitative and Mathematical Sciences

AS.110.791. Seminar in Analysis and Partial Differential Equations.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.

Area: Quantitative and Mathematical Sciences

AS.110.793. Seminar in Topology.

For graduate students only. Presentations of current research papers by faculty, graduate students and invited guest speakers.

Area: Quantitative and Mathematical Sciences

AS.110.794. Seminar in Category Theory.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.

Area: Quantitative and Mathematical Sciences

AS.110.795. Data Science Seminar.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.

Area: Quantitative and Mathematical Sciences

AS.110.798. Seminar in Number Theory.

Presentations of current research papers by faculty, graduate students and invited guest speakers. For graduate students only.

AS.110.799. Seminar in Algebraic Geometry.

For graduate students only. Presentations of current research papers by faculty, graduate students and invited guest speakers.

AS.110.800. Independent Study-Graduates.

Area: Quantitative and Mathematical Sciences

AS.110.801. Thesis Research.